

Doppler-Shannon Association in Vehicular Networks: Going Beyond Closest Point Association Policies

Ashutosh Balakrishnan

(IMT-Telecom Paris and LINC'S)

Joint work with F. Baccelli, S. Jhavar, P. Martins

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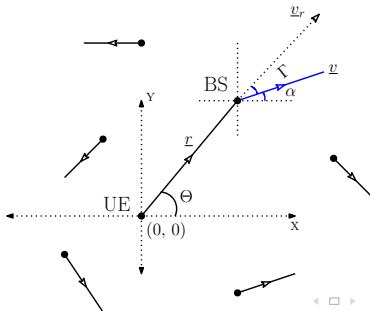
- 1 **Doppler aware** design and performance evaluation of vehicular/ non-terrestrial networks using stochastic geometry
 - Statistical characterization of the Doppler shift experienced at the UE (in 2D, 3D PPP)
 - Understanding time-selectivity (coherence time) of wireless channels
 - Doppler aware utilities
 - Doppler aware UE association framework (comparison with classical shannon based association)
 - Performance evaluation using stochastic geometry tools

Doppler Shift

- Doppler shift is the change in signal frequency due to relative motion of transmitter and receiver
- It is influenced by velocity along LoS $\underline{v}_r$
- Let Γ be the **relative angle between LoS and direction of motion (DoM)**, called as "*Doppler angle*"
- Doppler shift is defined as,

$$D = \frac{v_r}{\rho} = \frac{v \cos \Gamma}{\rho}$$

ρ : wavelength



Coherence time (Time selectivity)

- The effect of Doppler shift in frequency domain, is mapped in the time domain through coherence time
- Time duration for which the wireless channel remains “flat”, i.e., statistically i.i.d. In this duration, the wireless link is “stable” (slow fading)

$$T_c(\Gamma) = \frac{\mu}{|D|} = \frac{\mu\rho}{v \cos \Gamma}$$

μ is an engineering constant

- With random BS locations, Γ is random, D is random, and so is T_c
- In vehicular networks: the coherence time can be no longer considered constant, it is a random variable
- Hence, an adaptive coherence time framework, capturing the Doppler effect, is needed to be studied (UE adapts coherence time to associate with optimal BSs)

$$T_C(\Gamma) = \frac{\mu}{|D|} = \frac{\mu\rho}{v\cos\Gamma} = \frac{\mu c}{vf_c \cos\Gamma}$$

- Associating with a BS having **lower coherence time** affects system performance
 - $\cos\Gamma \in [-1, 1], D \in \left[\frac{-v}{\rho}, \frac{v}{\rho}\right]$
 - Doppler shift $\rightarrow \frac{v}{\rho}$ (max.) $\implies \Gamma \rightarrow 0$
 - BS located closer to the x-axis
 - **Higher f_c : lower coherence time** (high frequency bands cannot support high mobility)
 - **Higher v : lower coherence time**
- What about associating with a BS having minimum magnitude of Doppler?
 - Doppler shift $\rightarrow 0, \implies \Gamma \rightarrow \frac{\pi}{2}$
 - BS is located close to the zenith of the UE
 - But now $T_c \rightarrow \infty$ (we will see later the effect on system design)

2D PPP setting

- Static user equipment (UE) at the origin
- Mobile base station (BSs), PPP in $\mathbb{R}^2$
- Direction of Motion: i.i.d $\underline{v}$, angle α
- LoS between UE-BS, $\underline{r}$, angle Θ
- $\Theta \sim U[0, 2\pi)$ independent to $\alpha \sim U[0, 2\pi)$

$$\Gamma = (\Theta - \alpha) \bmod 2\pi \stackrel{d}{\sim} U[0, 2\pi),$$
$$f_D(x) = \frac{1}{\pi} \left((v/\rho)^2 - x^2 \right)^{-1/2}, \forall x \in \left[\frac{-v}{\rho}, \frac{v}{\rho} \right]$$

The Doppler shift is **arc-sine distributed** in 2D

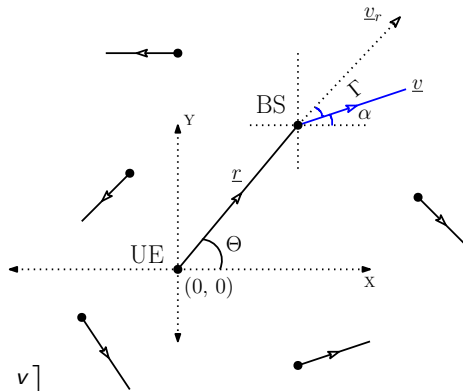
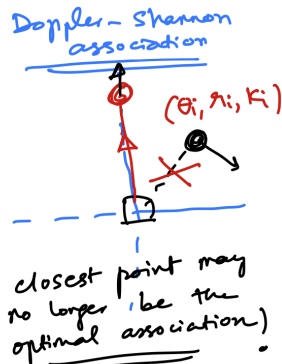
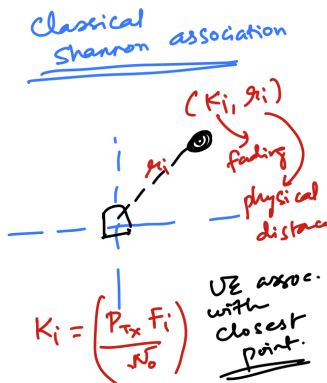


Figure: Considered 2D system model.

Doppler aware UE association policies

- UE is static at the origin
- Poisson distributed points (mobile BS nodes), X_i
- $X_i = \{r_i, \underbrace{\Theta_i, \alpha_i}_{\Gamma_i}\}$
- K_i : channel fading gain
- **Classical Shannon association:** $f(r_i, K_i)$
- **Doppler-Shannon association:** $f(r_i, \Gamma_i, K_i)$
- r_i : UE-BS Euclidean distance



Doppler aware utilities: Utility 1 [Data reception]

- To capture the data reception (in bits) at the UE from a BS i :

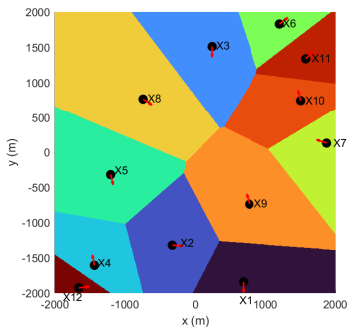
$$U_i(\Gamma, R, K) \triangleq T_c(\Gamma_i) B \log_2(1 + K_i R_i^{-\sigma})$$

σ : path loss exponent

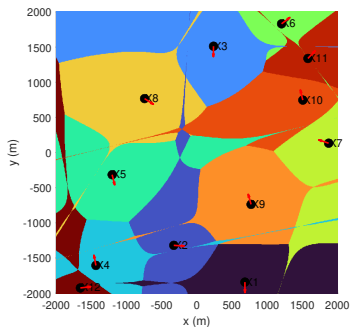
- We consider the coverage region of a BS at X_i as the set of locations on $\mathbb{R}^d$, as

$$\mathcal{R}_i := \{z \in \mathbb{R}^d : U_i(z) > U_j(z) \text{ for all } j \neq i\}.$$

- Here, if $\Gamma_i \rightarrow \frac{\pi}{2} \implies D(\Gamma_i) \rightarrow 0 \implies T_c(\Gamma_i) \rightarrow \infty \implies U_i \rightarrow \infty$
- $U_i \rightarrow \infty$ results in **very elongated tessellations** perpendicular to the direction of motion $\Gamma_i \rightarrow \frac{\pi}{2}$



(a)



(b)

Figure: (a) Classical Shannon association $U_i = B \log_2(1 + K_i R_i^{-\sigma})$, (b) In presence of Doppler shift through $U_i = T_c(\Gamma_i) B \log_2(1 + K_i R_i^{-\sigma})$. Observe the non-convexity of the tessellations

- Note that we perform analysis in a time-snapshot (at a very small time interval δt)
- Hence, infinitely long “rays” varying in a time dynamical system is a challenge
- We limit the extent of elongation of the tessellation “rays”, through a **limiting factor**

$$U_{1,i} \triangleq \min\{T_c(\Gamma_i), \zeta\} B \log_2(1 + K_i R_i^{-\sigma}),$$

- $\zeta > 0$ is the limiting factor: it constraints $T_{c_i} \rightarrow \infty$
- Very small $\zeta \sim 10^{-3}$ results in the tessellations being convex again

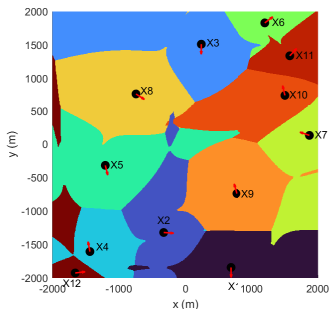


Figure: Tessellations with $\zeta = 0.005$ obtained from association through first utility

Utility-II [Data rate]

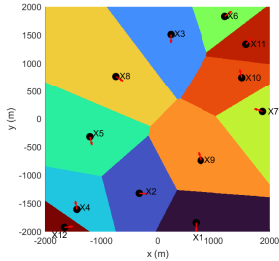
- Consider a TDMA system, having ε channel estimation time and remaining $(T_c - \varepsilon)$ data transmission time
- The second utility U_2 , captures data rate (in [bps]) obtained at a typical UE from a BS at X_j . The utility is defined as,

$$U_{2,i} := U_2(\Gamma_i, R_i, K_i) \triangleq \left(1 - \frac{\varepsilon}{T_c(\Gamma_i)}\right)^+ B \log_2(1 + K_i R_i^{-\sigma}),$$

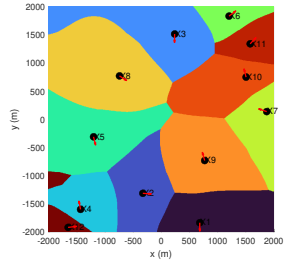
where $|X| = R$ and ε is the channel estimation time

- Note: channel estimation time cannot exceed the coherence time ($T_c > \varepsilon$), else negative utility

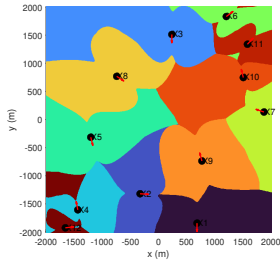
$$\begin{cases} \text{if } \varepsilon \ll T_c & \text{the effect of Doppler vanishes,} \\ \text{if } \varepsilon < T_c & \text{the effect of Doppler is prominent} \end{cases}$$



(a)



(b)



(c)

Figure: (a) Very efficient channel estimation $\varepsilon = 10^{-6}$, (b) $\varepsilon = 5 \times 10^{-5}$, (c) $\varepsilon = 10^{-4}$

Doppler-shannon: 2D Performance Evaluation

- The objective: to associate the UE with the **max. utility BS** (not closest)

$$x^* = \arg \max_i U_i$$

- For independent, Θ, α : $\Gamma(\Theta, \alpha) = \cos(\Theta - \alpha) \stackrel{d}{\sim} \cos \Theta \equiv \Gamma(\Theta)$
- Hence, $U(\Gamma, R, K) \equiv U(\Theta, R, K)$

Theorem

The coverage probability at a typical UE, to meet the user QoS τ , is given by

$$\mathbb{P}\left(\max_i U(\Theta_i, R_i, K_i) \geq \tau\right) = 1 - e^{-\lambda \pi \mathbb{E}_K \left[K^{\frac{2}{\sigma}} \right] \mathbb{E}_{\Theta} [g(\Theta, \tau)^2]}.$$

where $g(\Theta, \tau)$, represents a cut-off amoeba function; σ is path loss exponent.

$$\text{For } U_1, g(\Theta, \tau) := \left(2^{\tau / \min\{T_c(\Theta), \zeta\}B} - 1 \right)^{-\frac{1}{\sigma}}$$

$$\text{For } U_2, g(\Theta, \tau) := \left(2^{\tau / \left(1 - \frac{\epsilon}{T_c(\Theta)}\right)^+ B} - 1 \right)^{-\frac{1}{\sigma}}$$

Here, $T_c(\Theta) = \frac{\mu}{|D(\Theta)|}$. Hence, the Doppler magnitude (coming from the angular location of the BSs) influences the shape of tessellations.

Comparison with classical Shannon association

- The nearest point distribution is

$$f_{R_o}(r) := 2\lambda\pi r e^{-\lambda\pi r^2}$$

- The outage probability is

$$\mathbb{P}[U(\Theta_o, R_o, K_o) \leq \tau] = \mathbb{E}_{\Theta_o, K_o} \left[e^{-\lambda\pi K_o^{2/\sigma} g(\Theta_o, \tau)^2} \right]$$

- Comparing D-S and Shannon performance by Jensen's inequality, we have:

$$\underbrace{e^{-\lambda\pi \mathbb{E}_{\Theta, K} [K^{2/\sigma} g(\Theta, \tau)^2]}}_{\text{Outage in Doppler-Shannon association}} \leq \underbrace{\mathbb{E}_{\Theta, K} \left[e^{-\lambda\pi K^{2/\sigma} g(\Theta, \tau)^2} \right]}_{\text{Outage in Shannon association}}$$

- In other words, comparing the coverage probabilities we have

$$\mathbb{P} \left(\max_i U(\Theta_i, R_i, K_i) > \tau \right) \geq \mathbb{P} (U(\Theta_o, K_o, R_o) > \tau)$$

Network Performance: Utility 1

$$U_{1,i} \triangleq \min\{T_c(\Gamma_i), \zeta\} B \log_2(1 + K_i R_i^{-\sigma}),$$

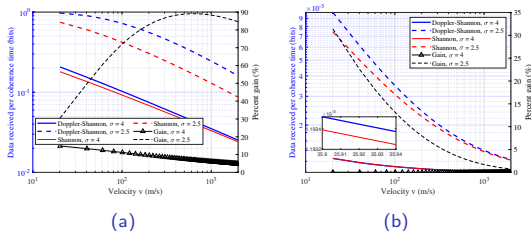
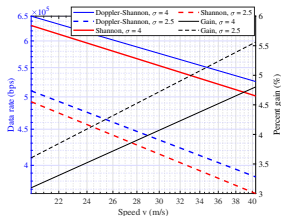


Figure: Performance evaluation of policy 1 against velocity in a 2D PPP setting at (a) $\lambda = 10^{-3}$ (b) $\lambda = 10^{-6}$

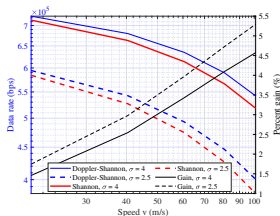
- Performance of D-S association $\geq$ the Shannon association performance
- Higher gains observed at moderate λ
- As λ decreases, gains decrease, D-S $\approx$ Shannon

Network Performance: Utility 2

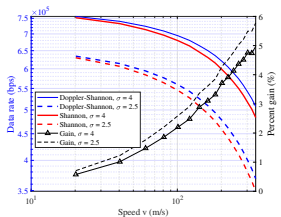
$$U_2(\Gamma_i, R_i, K_i) \triangleq \left(1 - \frac{\varepsilon}{T_c(\Gamma_i)}\right)^+ B \log_2(1 + K_i R_i^{-\sigma})$$



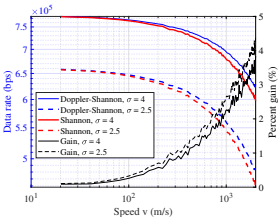
(a)



(b)



(c)



(d)

Figure: Performance of Utility 2, with PLE $\sigma = \{2.5, 4\}$ at (a)

$\varepsilon = 8 \times 10^{-4}, \lambda = 1$, (b) $\varepsilon = 3 \times 10^{-4}, \lambda = 1$, (c) $\varepsilon = 10^{-4}, \lambda = 1$, (d)

$\varepsilon = 10^{-5}, \lambda = 1$.

- From (a) - (d), the channel estimation time decreases.
- The norm of velocity supported by the system: up to 40 mps in (a) and keeps increasing around 1000 mps (d)
- The gain in the D-S association w.r.t. Shannon association also decreases with lower channel estimation time interval

3D PPP setting

- BSs modeled in 3D as PPP
- LoS: Elevation $\Phi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, Azimuth $\Theta \in [0, 2\pi)$
- DoM: Elevation $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, Azimuth $\beta \in [0, 2\pi)$
- The angles are independent to each other
- The cosine of the Doppler angle Γ is
$$\cos \Gamma = \cos \Phi \cos \alpha \cos(\Theta - \beta) + \sin \Phi \sin \alpha$$
- The PDF of the elevation angles:
$$f_{\Phi}(\phi) = \frac{1}{2} \cos \phi \text{ and } f_{\alpha}(a) = \frac{1}{2} \cos a$$
- The PDF of azimuth angles are distributed as $U[0, 2\pi)$

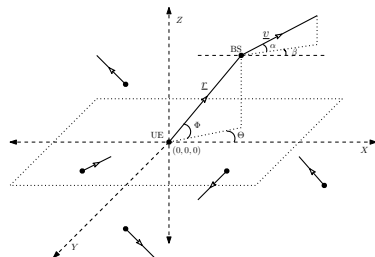


Figure: 3D geometry

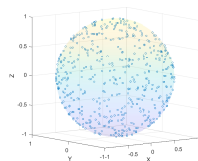


Figure: Illustration of 3D sampling of points for a given LoS norm r , $f_{\Phi} = \frac{1}{2} \cos \phi$, $\theta \sim [0, 2\pi)$

3D Doppler Spectrum

- In the 3D scenario, the Doppler shift is uniformly distributed:

$$\cos \Gamma \stackrel{d}{\sim} \mathcal{U}[-1, 1] \text{ and } D \stackrel{d}{\sim} \mathcal{U}\left[\frac{-v}{\rho}, \frac{v}{\rho}\right].$$

- The Doppler angle Γ is independent of the DoM angle α, β , i.e., $\Gamma(\Theta, \Phi, \alpha, \beta) \stackrel{d}{\sim} \Gamma_1(\Theta, \Phi)$

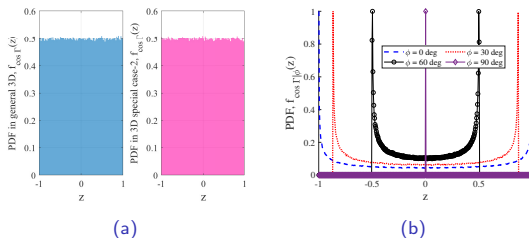
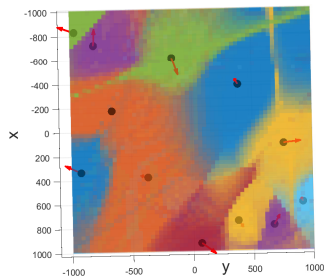


Figure: (a) Simulated PDF of $\cos \Gamma$ in generalized 3D case, and its comparison with $\cos \Gamma_1$, (b) Simulation of the conditional PDF of $f_{\cos \Gamma | \Phi = \phi}(\gamma)$ (2D spectrum)

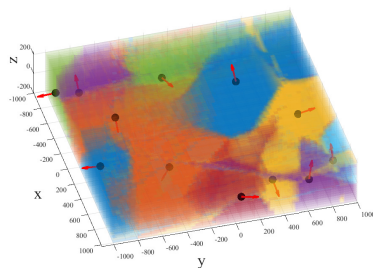
- Conditioning the 3D Doppler with respect to elevation, gives 2D Doppler (Fig. (b))

3D tessellations: Utility 1

$U_i = T_c(\Gamma_i)B \log_2(1 + K_i R_i^{-\sigma})$ [bits] (illustrating tessellations without the limiting factor)



(a)

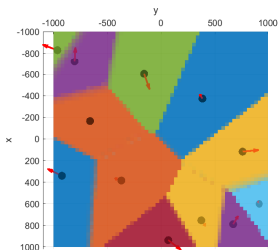


(b)

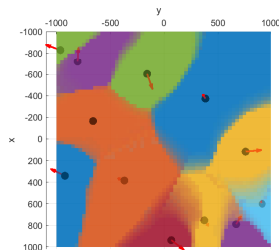
Figure: 3D tessellations originating from associations through policy 1 (a) top view, (b) side view

3D tessellations: Utility 2

$$U_2(\Gamma_i, R_i, K_i) \triangleq \left(1 - \frac{\varepsilon}{T_c(\Gamma_i)}\right)^+ B \log_2(1 + K_i R_i^{-\sigma})$$



(a)



(b)

Figure: 3D tessellations (top views) originating from associations through policy 2 (a) very efficient channel estimation (effect of Doppler not visible), (b) higher channel estimation time (effect of Doppler on the tessellations is clearly visible)

Doppler-Shannon outage in 3D

We have, in 3D setting: $U(\Gamma, R, K) \equiv U(\Phi, \Theta, R, K)$

Theorem

For mobile nodes modeled spatially as a 3D Poisson point process, the outage probability at a typical UE to meet the user QoS τ is given by

$$\mathbb{P}\left(\max_i U(\Theta_i, \Phi_i, R_i, K_i) < \tau\right) = e^{-\frac{4\lambda\pi}{3} \mathbb{E}_{\Phi, \Theta, K} \left[K^{\frac{3}{\sigma}} g(\Theta, \Phi, \tau)^3 \right]}$$

with $g(\Theta, \phi, \tau)^3$ representing the cut-off amoeba function

The cut-off functions $g(\Theta, \phi, \tau)$ for the first utility U_1 :

$$g(\Theta, \phi, \tau) = \left(2^{\tau / \min\{T_c(\Theta, \phi), \zeta\} B} - 1 \right)^{-\frac{1}{\sigma}}$$

For the second utility U_2 , the cutoff function is

$$g(\Theta, \phi, \tau) = \left(2^{\tau / \left(1 - \frac{\xi}{T_c(\Theta, \phi)}\right)^+ B} - 1 \right)^{-\frac{1}{\sigma}}$$

Comparison with Classical Shannon Association

Let R_o be the distance to the nearest BS. Then R_o follows a 3D analogue of the Rayleigh distribution with density given, for $r \geq 0$, by

$$f_{R_o}(r) = 4\pi\lambda r^2 e^{-\frac{4\pi\lambda}{3}r^3}$$

The outage probability based on nearest point association is

$$\mathbb{P}(U(\theta_o, \phi_o, K_o, r_o) \leq \tau) = \mathbb{E}_{\theta_o, \phi_o, K_o} \left[e^{-\frac{4\pi\lambda}{3} K_o^{\frac{3}{\sigma}} g(\theta_o, \phi_o, \tau)^3} \right]$$

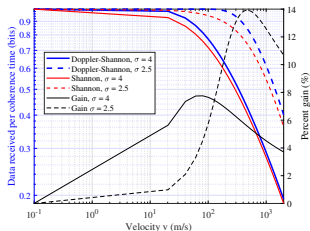
Comparing the outage probabilities using the Jensen's inequality.

$$e^{-\frac{4\pi\lambda}{3} \mathbb{E}_{\phi, \Theta, K} [K^{\frac{3}{\sigma}} g(\Theta, \phi, \tau)^3]} \leq \mathbb{E}_{\Theta, \phi, K} \left[e^{-\frac{4\pi\lambda}{3} K^{\frac{3}{\sigma}} g(\Theta, \phi, \tau)^3} \right]$$

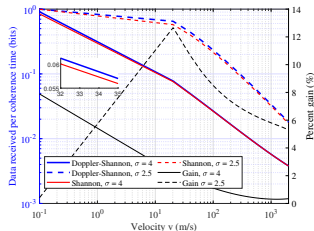
or,

$$\mathbb{P}\left(\max_i U_i(\theta_i, \phi_i, R_i, K_i) < \tau\right) \leq \mathbb{P}(U(\theta_o, \phi_o, K_o, r_o) \leq \tau)$$

Network performance: Utility 1



(a)

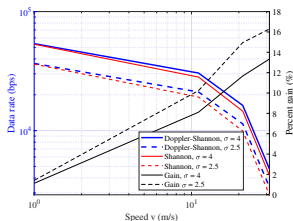


(b)

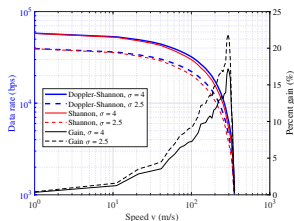
Figure: Performance evaluation of Policy 1 against velocity under 3D PPP setting with PLE $\sigma = \{2.5, 4\}$ at (a) $\lambda = 10^{-2}$ and (b) $\lambda = 10^{-4}$.

Network Performance: Utility 2

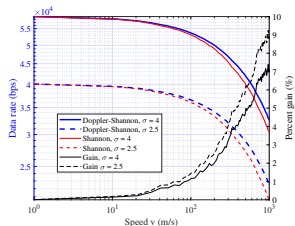
$$U_2(\Gamma_i, R_i, K_i) \triangleq \left(1 - \frac{\varepsilon}{T_c(\Gamma_i)}\right)^+ B \log_2(1 + K_i R_i^{-\sigma}),$$



(a)



(b)



(c)

Figure: Performance evaluation of policy 2 against velocity under 3D PPP setting at (a) $\varepsilon = 10^{-3}$, $\lambda = 1$, (b) $\varepsilon = 10^{-4}$, $\lambda = 1$, (c) $\varepsilon = 10^{-5}$, $\lambda = 1$

NTN Simulator Development

- 1 Integrate all the available open source information (like satellite TLE data, satellite orbital propagator, earth rotation) in a single platform.
- 2 See the evolution of any LEO satellite in real time.
- 3 The constellation which we currently study are IRIDIUM satellites.
- 4 Aim to develop a dimensioning & planning tool, which can be used for routing studies as well.
- 5 Implementation by Shankhadip Mondal (M2, Telecom Paris)

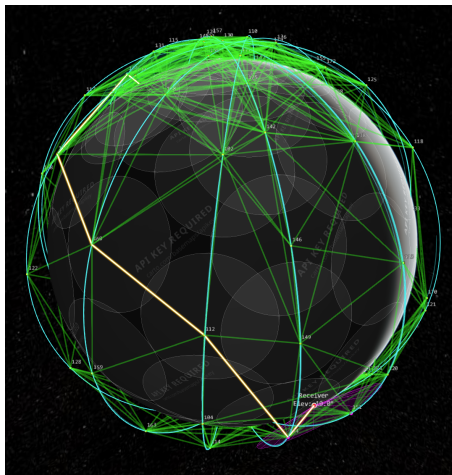


Figure: NTN real time simulator development

https://www.dropbox.com/scl/fi/a7k3fhw4a5rcqz0uxpst/dynamic_routes.mp4?rlkey=7f8hoi89948qzz0laiwqby29b&st=afdy5qcu&dl=0

- 1 NTN simulator development (with P. Martins)
- 2 Routing protocols in NTN (with P. Martins)
- 3 Channel modeling in NTN (IEEE SA P1944) (with F. Baccelli, P. Martins, M.-S. Alouini)
- 4 Quantifying SNR gain with jointly visible RISs (with Junse Lee, F. Baccelli)
- 5 Doppler-Shannon framework analysis in a Poisson Line Cox Process (with S. Jhavar, F. Baccelli, P. Martins).

Thank you!

Questions, Suggestions

A. Balakrishnan
(balakrishnan@telecom-paris.fr)

A. Balakrishnan, J. S. Lee, F. Baccelli, “[Joint visibility analysis of RIS in non-terrestrial networks through stochastic geometry](#),” (under review, IEEE Trans. Wireless Commun., June 2026)

A. Balakrishnan, F. Baccelli, S. Jhavar, P. Martins, “[Doppler-Shannon association in vehicular networks: A stochastic geometry analysis](#),” (under review, IEEE Trans. Wireless Commun., April 2026)

A. Balakrishnan, P. Popineau, and P. Martins, “[Timing advance estimation in LEO satellite networks: A recursive Bayesian study](#)” accepted in *Proc. IEEE GLOBECOM*, Taiwan, Dec. 2025. 