



PROGRAMME
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Institut de Recherche
en Informatique de Toulouse



Chirp Based Waveform Design for Next Generation Non Terrestrial Networks

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NTN Days
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Outline

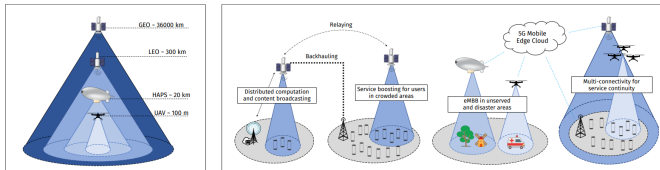
- 1 Context
- 2 Spectral Analysis of the AFDM Modulation
- 3 LoRa-Based Chirp Division Multiplexing
- 4 Future Works

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Broadband Communications (6G)

- Next-generation networks (6G) demand **higher data rates, global 3D connectivity, integrated sensing and communication (ISAC),** and a **unified architecture**.
- Non-Terrestrial Networks (NTN) meet these demands but introduce many challenges, primarily **high mobility**.¹

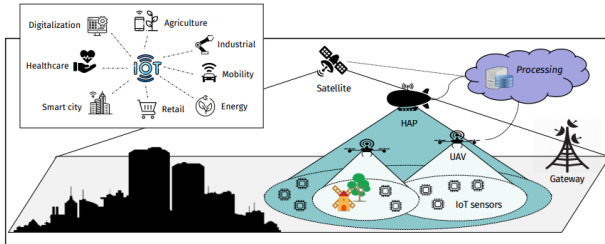


- **The Need for Resilient Waveforms** to ensure high spectral efficiency, robust communication links, and reliable sensing under severe NTN channel conditions.

¹Image credit: Giordani et al *Non-terrestrial networks in the 6G era: Challenges and opportunities*, IEEE Network.

Satellite IoT (DtS IoT)

- Global IoT scaling drives the demand for **ubiquitous coverage** and **higher data rates** without sacrificing long range capabilities.
- NTN enables Direct-to-Satellite (DtS) IoT, significantly extending network reach and capacity. ²



- **Waveform Design:** Developing solutions able to support massive IoT expansion and dense, simultaneous device transmissions.

²Image credit: Wang et al *On the performance of non-terrestrial networks to support the internet of things*, Asilomar.

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Motivation

- Beyond 5G and 6G systems require the support of high mobility applications.
- The AFDM alternative: A chirp-based multicarrier waveform.
- Although performance gains in power efficiency are well studied, the spectral characteristics of AFDM remain under-explored. In this work we ³:
 - Provide a rigorous analytical derivation of the continuous-time AFDM signal . . .
 - and its power spectral density (PSD).
 - Study the influence of different parameters, notably the chirp rate, on out-of-band (OOB) emissions.

³El Marzougui, Omar and Thomas, Nathalie and Poulliat, Charly, *Spectral Analysis Of The AFDM Modulation*, 34th European Signal Processing Conference (EUSIPCO 2026).

Signal Model

- AFDM transmits symbols using a bank of chirps performing multiple frequency sweeps over the interval $[0, B]$. It is characterized by an integer C , representing the number of full frequency sweeps.
- For a set of N chirps over a duration $T = N/B$, the instantaneous frequency of the k -th chirp is given by the following formula, with $0 \leq t \leq T$, $0 \leq k \leq N - 1$ and $1 \leq l \leq C$:

$$f_k(t) = \frac{B}{N}k + 2c_1 B^2 t - B \sum_{l=1}^C \mathcal{H}(t - \tau_{l,k}), \quad c_1 = \frac{C}{2N}, \quad \tau_{l,k} = \frac{Nl - k}{2Nc_1 B}$$

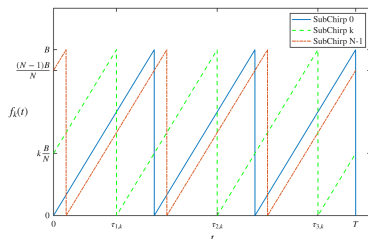
- The chirp rate is adjusted to the Doppler such that: $c_1 = \frac{2\lceil f_{\max} T \rceil + 1}{2N}$.
- $\tau_{l,k}$ are the frequency reset times to 0, temporally spaced by $T_C = \tau_{l+1,k} - \tau_{l,k} = \frac{T}{C}$.

Signal Model

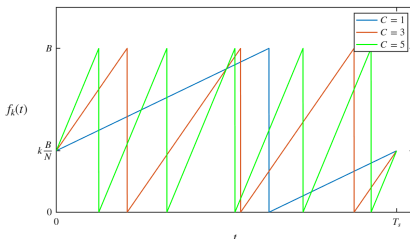
- The phase of the AFDM chirp is expressed as follows:

$$\phi_k(t) = 2\pi \left[\frac{B}{N}kt + c_1 B^2 t^2 - B \sum_{l=1}^C (t - \tau_{l,k}) \mathcal{H}(t - \tau_{l,k}) + c_2 k^2 \right]$$

AFDM Chirp Bank



Different Chirp Rates



- The complex envelope of the transmitted AFDM signal is:

$$x(t) = \sum_n s(t - nT)g_T(t - nT)$$

where $s(t)$ is the AFDM symbol: $s(t) = \sum_{k=0}^{N-1} s_k e^{j\phi_k(t)}$.

Analytical PSD Formula

- Assuming that the transmitted data symbols are statistically independent and identically distributed (i.i.d.) with zero mean, the cross-correlation terms cancel out. Thus, the PSD of the AFDM signal is defined by the sum of the power spectra of its individual sub-chirps:

$$P(f) = \frac{\sigma_s^2}{T} \sum_{k=0}^{N-1} |\Phi_k(f)|^2$$

Where $\Phi_k(f) = \int_0^T e^{j\phi_k(t)} e^{-j2\pi ft} dt$ is the Fourier transform of the sub-chirp $e^{j\phi_k(t)}$.

- We express the Fourier transform integral of the k -th sub-chirp for $0 \leq k \leq N-1$ and $1 \leq l \leq C$ as follows:

$$\Phi_k(f) = \int_0^T e^{j2\pi(\frac{B}{N}kt + c_1 B^2 t^2 - B \sum_{l=1}^C (t - \tau_{l,k}) \mathcal{H}(t - \tau_{l,k}) + c_2 k^2 - ft)} dt$$

Analytical PSD Formula

- The idea is to divide the integral over the intervals $[\tau_{l,k}, \tau_{l+1,k}]$, which will result in a sum of integrals, and to express each term of this sum using Fresnel integrals:

$$C(x) = \int_0^x \cos(u^2 \pi/2) du, \quad S(x) = \int_0^x \sin(u^2 \pi/2) du$$

- Let $Z(a, b, t_1, t_2)$ be the function defined by $Z(a, b, t_1, t_2) = \int_{t_1}^{t_2} \exp[j2\pi(at + bt^2)] dt$. This function can be expressed in terms of Fresnel functions:

$$Z(a; b; t_1; t_2) = \frac{1}{2\sqrt{b}} e^{-j2\pi \frac{a^2}{4b}} \left[F\left(2\sqrt{b}\left(t_2 + \frac{a}{2b}\right)\right) - F\left(2\sqrt{b}\left(t_1 + \frac{a}{2b}\right)\right) \right]$$

- Where $F(x) = C(x) + jS(x)$.

Analytical PSD Formula

- Finally, using these tools, we can deduce an analytical expression of the Fourier transform of an AFDM sub-chirp using the $Z(\cdot)$ function as follows:

$$\Phi_k(f) = \sum_{l=0}^C A_{k,l} \cdot Z \left(B \left(\frac{k}{N} - l \right) - f, c_1 B^2, \tau_{l,k}, \tau_{l+1,k} \right)$$

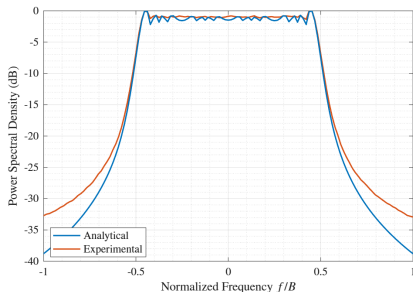
- Where $A_{k,l}$ is a phase coefficient independent of time t and which can be expressed as follows:

$$A_{k,l} = \exp \left[j2\pi \left(\frac{Nl(l+1)}{2C} - \frac{kl}{C} + c_2 k^2 \right) \right]$$

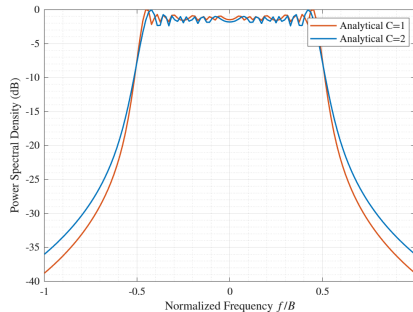
Numerical Results

- **Impact of the chirp rate:** A higher chirp rate leads to more significant out-of-band (OOB) emissions. This highlights a trade-off between Doppler robustness and spectral containment.

Analytical vs Experimental.



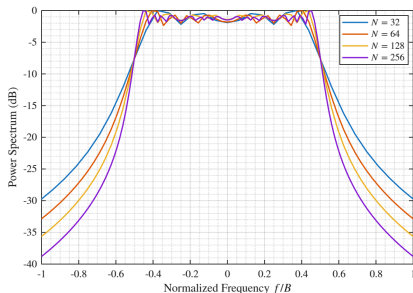
Influence of the Chirp Rate.



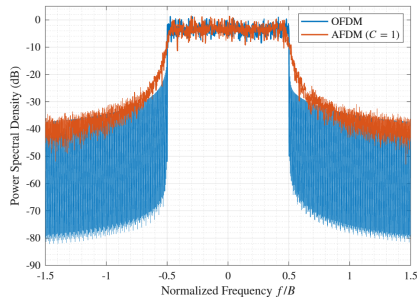
Numerical Results

- **Impact of sub-chirps (N):** Increasing the number of sub-chirps leads to a more compact spectrum, which translates into lower OOB emissions.
- **AFDM vs OFDM:** When using a rectangular window, the AFDM spectrum exhibits more OOB emissions compared to OFDM.

Influence of the N° of Sub-chirps.



AFDM vs OFDM.





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Thank you for your attention !



Questions?
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