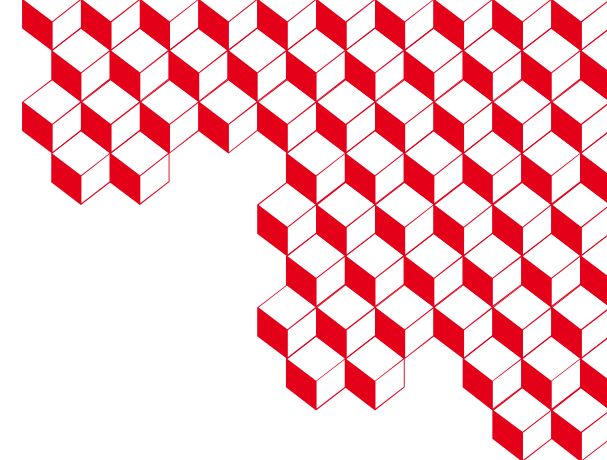




Equalization for Delay-Doppler Modulations in mobile and NTN channels

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CEA Leti DSYS/STSF/LS2PR



Context: Doppler-Robust Waveforms

1

Scope

We study delay-Doppler (DD) waveforms, OTFS and ODDM, for mobile channels, with applications to mobile 6G and Non-Terrestrial Networks (NTN).

2

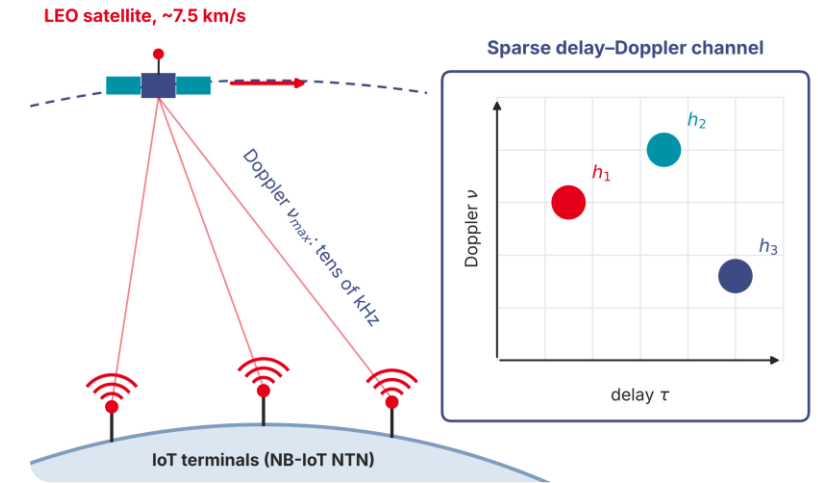
Why Doppler is a problem

High-speed vehicles, UAVs and LEO satellites induce large, fast-varying Doppler shifts. They break the subcarrier orthogonality of OFDM, so communication systems must be made robust to strong Doppler.

3

Why delay-Doppler waveforms

The doubly-dispersive channel is sparse and quasi-static in the DD domain, so each path is resolved in delay and Doppler. OTFS and ODDM are therefore potential solutions.



444 Hz

Terrestrial vehicular, 120 km/h at 4 GHz

15–30 kHz

LEO NTN, L/S bands: $\approx 35\text{--}70\times$ larger Doppler

This talk: low-complexity and turbo equalization for OTFS / ODDM.

Outline

1. Delay-Doppler Modulation and Channel Model

2. State of the Art: Detection Algorithms

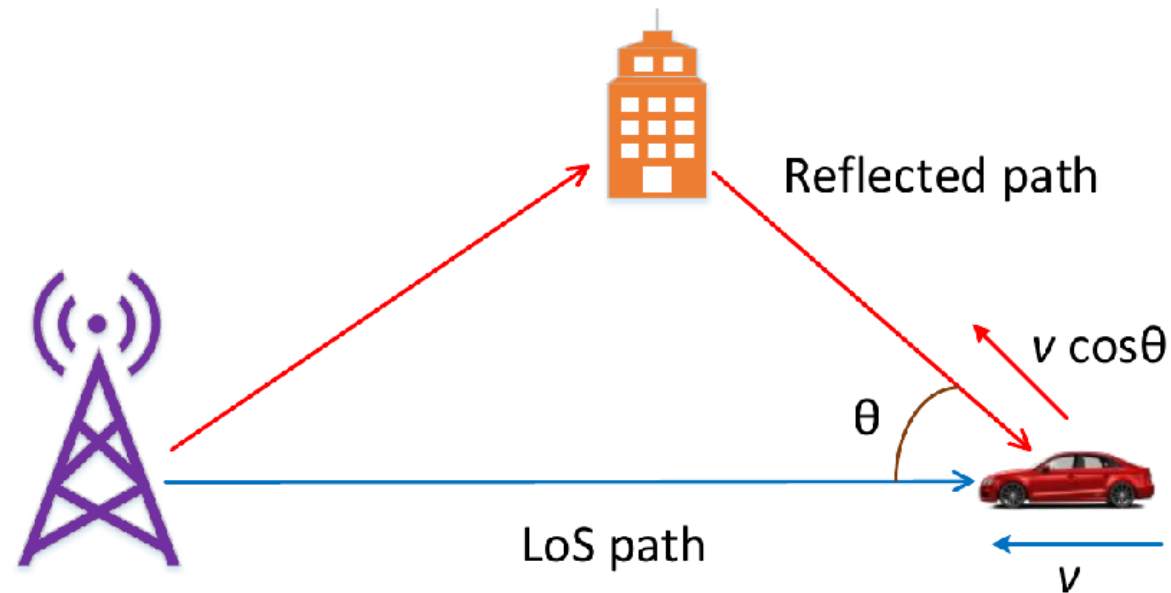
LMMSE, Message Passing, MRC/Rake, Conjugate gradient and Sparse Preconditioned CG

3. Block LMMSE Turbo Equalization

4. Simulation Results

5. Conclusions

OTFS / ODDM – Delay-Doppler Modulation



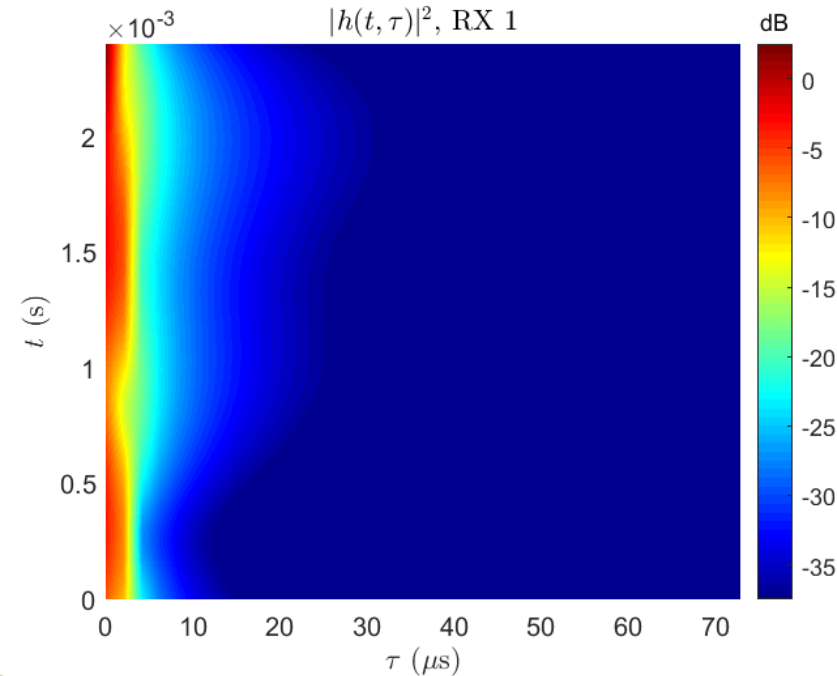
- Doppler frequency of LoS path: $\nu_1 = f_c \frac{v}{c}$
- Doppler frequency of reflected path: $\nu_2 = f_c \frac{v \cos \theta}{c}$
- Doppler spread: $\nu_2 - \nu_1$

$$\text{TX: } s(t) \quad \text{RX: } r(t) = h_1 s(t - \tau_1) e^{-j2\pi\nu_1 t} + h_2 s(t - \tau_2) e^{-j2\pi\nu_2 t}$$

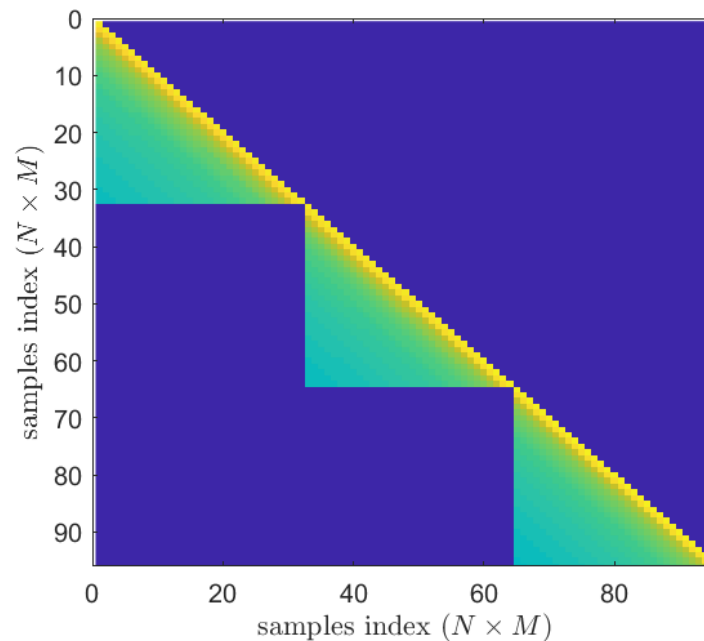
The channel is characterized by parameters (h_i, τ_i, ν_i) complex coefficients associated with their delays and Doppler shifts

Channel Examples

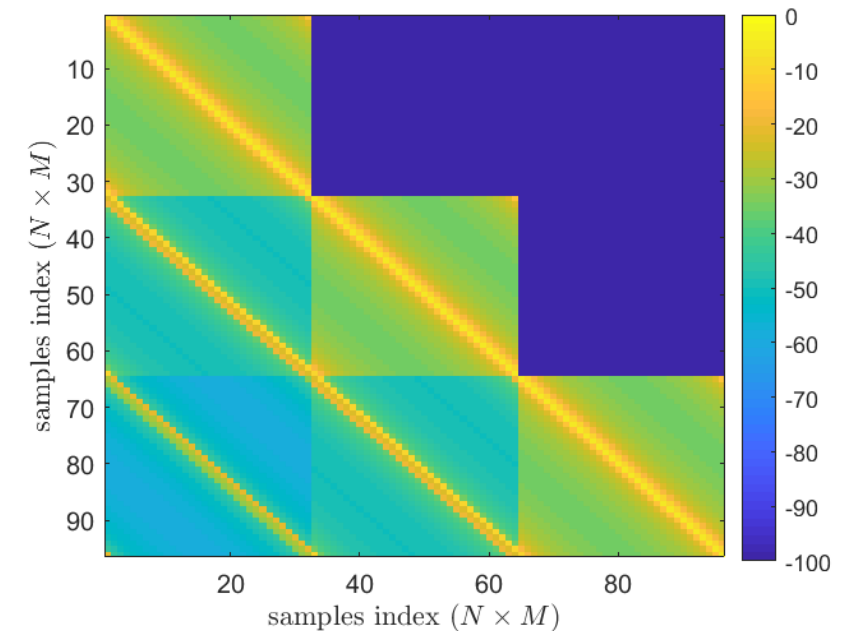
- Extended vehicular channel A (EVA) : $g(t, \tau) = \sum_i g_i \delta(\tau - \tau_i) \times \exp j2\pi v_i t$
 - $\tau_i = [0, 30, 150, 310, 370, 710, 1090, 1730, 2510] \times 10^{-9} s$
 - $v_i = v_{max} \cos(\theta_i)$ avec $\theta_i \sim \mathcal{U}(0, 2\pi)$ with $v_{max} = V_{max} \times \frac{f_c}{c} = 444,4 Hz$ (120 km/h @ 4GHz)
 - g_i complex gaussian such as $\mathbb{E}[|g_i|^2] = [0, -1.5, -1.4, -3.6, -0.6, -9.1, -7.0, -12.0, -16.9]$ in dB



1 channel impulse response realization



Channel matrix G (time domain,
guard intervals removed)



Delay-Doppler channel matrix H

Detection Algorithms

In delay-Doppler, vectorized form:

$$\mathbf{y} = \mathbf{H} \cdot \mathbf{x} + \mathbf{z},$$

with $\mathbf{H} = (\mathbf{I}_M \otimes \mathbf{F}_N) \cdot (\mathbf{P}^T \cdot \mathbf{G} \cdot \mathbf{P}) \cdot (\mathbf{I}_M \otimes \mathbf{F}_N^\dagger) \in \mathbb{C}^{NM \times NM}$

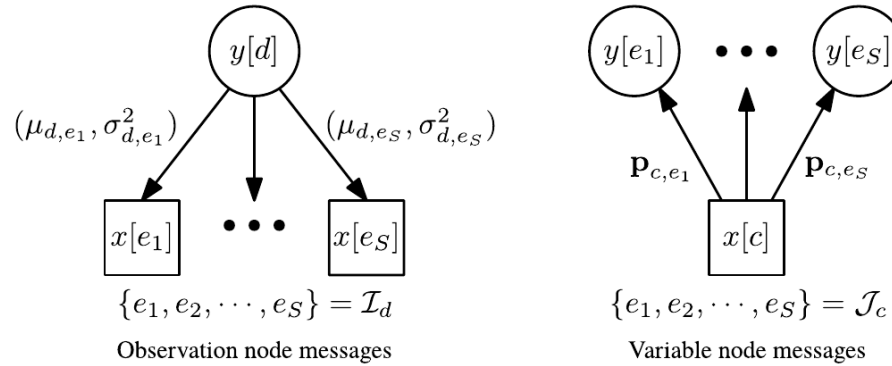
LMMSE :

$$\hat{\mathbf{x}} = (\mathbf{H}^\dagger \cdot \mathbf{H} + \sigma_w^2 \mathbf{I}_{MN})^{-1} \cdot \mathbf{H}^\dagger \cdot \mathbf{y}.$$

Complexity: matrix inversion of size $MN \rightarrow O(M^3 N^3)$

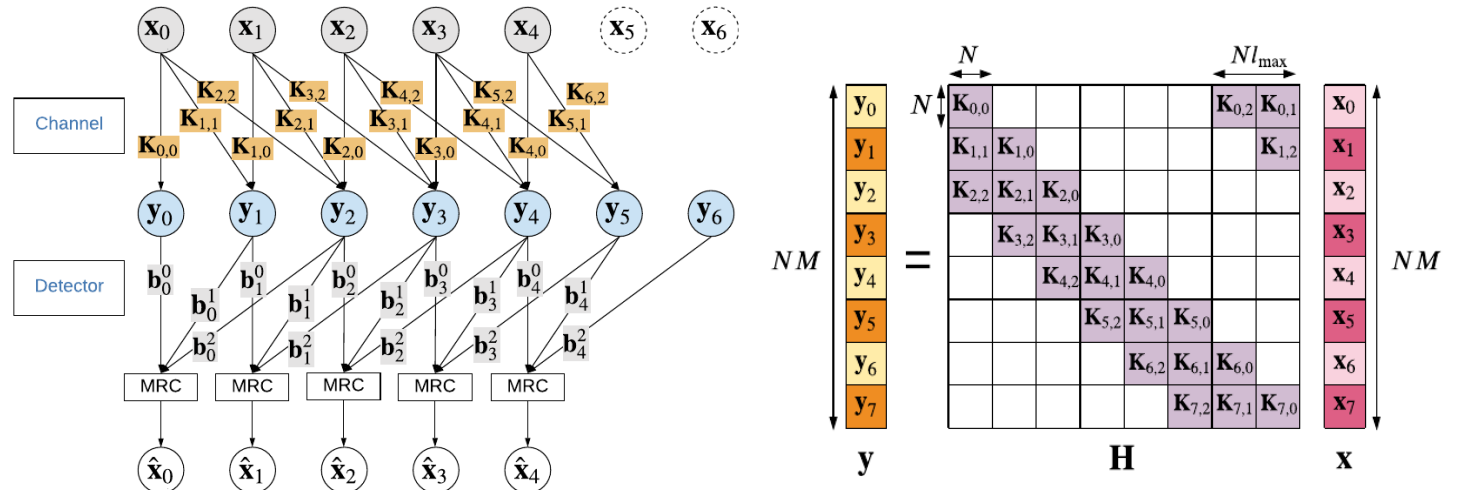
Message Passing Algorithm (MPA)

- Assumes sparsity in delay-Doppler
- Otherwise complex
- $\rightarrow O(n_{ite} \times MN \times S \times |\mathcal{A}|)$

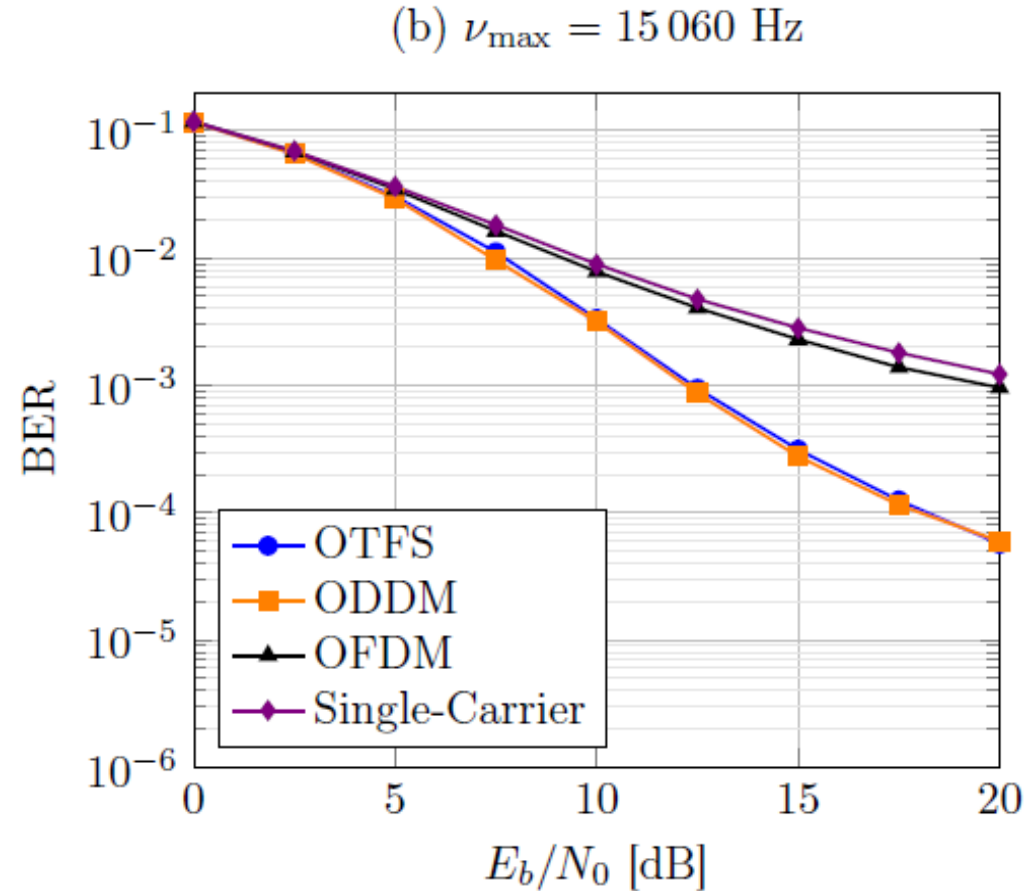
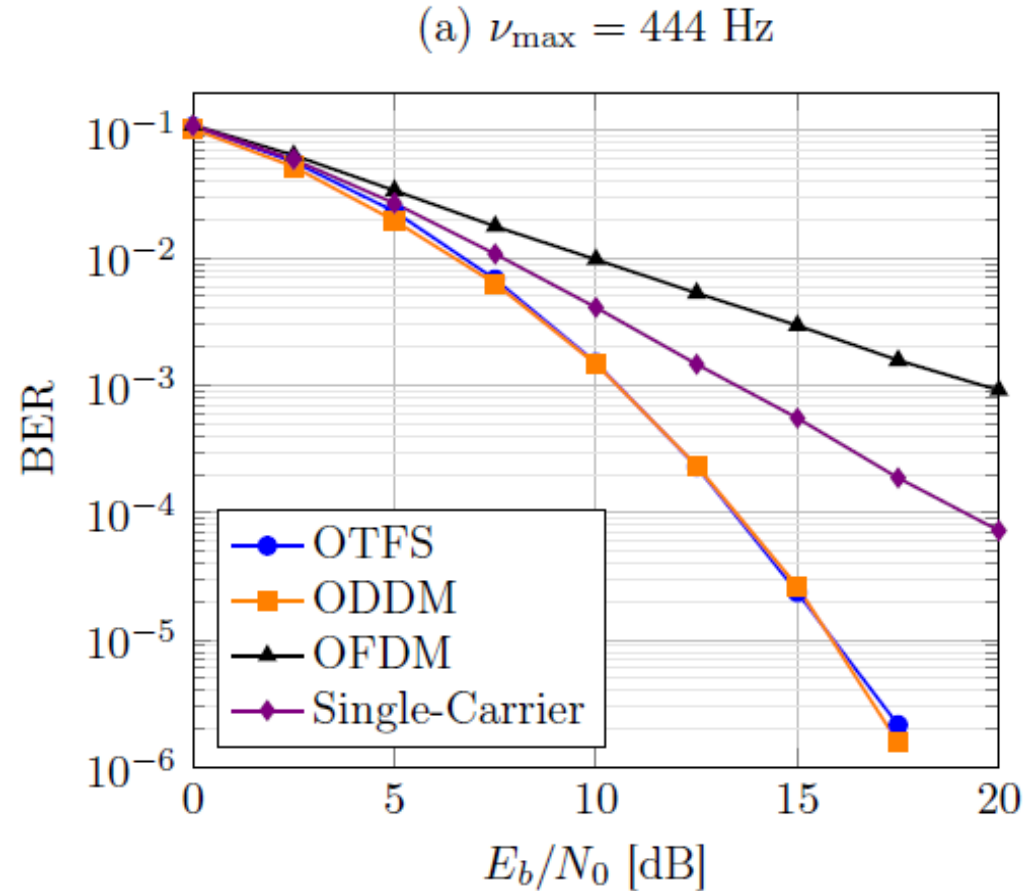


Maximal Ratio Combiner

- SIC on decided or equalized symbols
- Poor performance when $N \times M$ is small
- $O(n_{ite} \times MN \times L)$



Waveforms comparison with LMMSE equalizer



$M=N=16$
 $\Delta f = 15$ kHz
 $\sigma_\tau = 1\mu\text{s}$

Same parameters for all modulations,

Known channel + LMMSE eqlzr

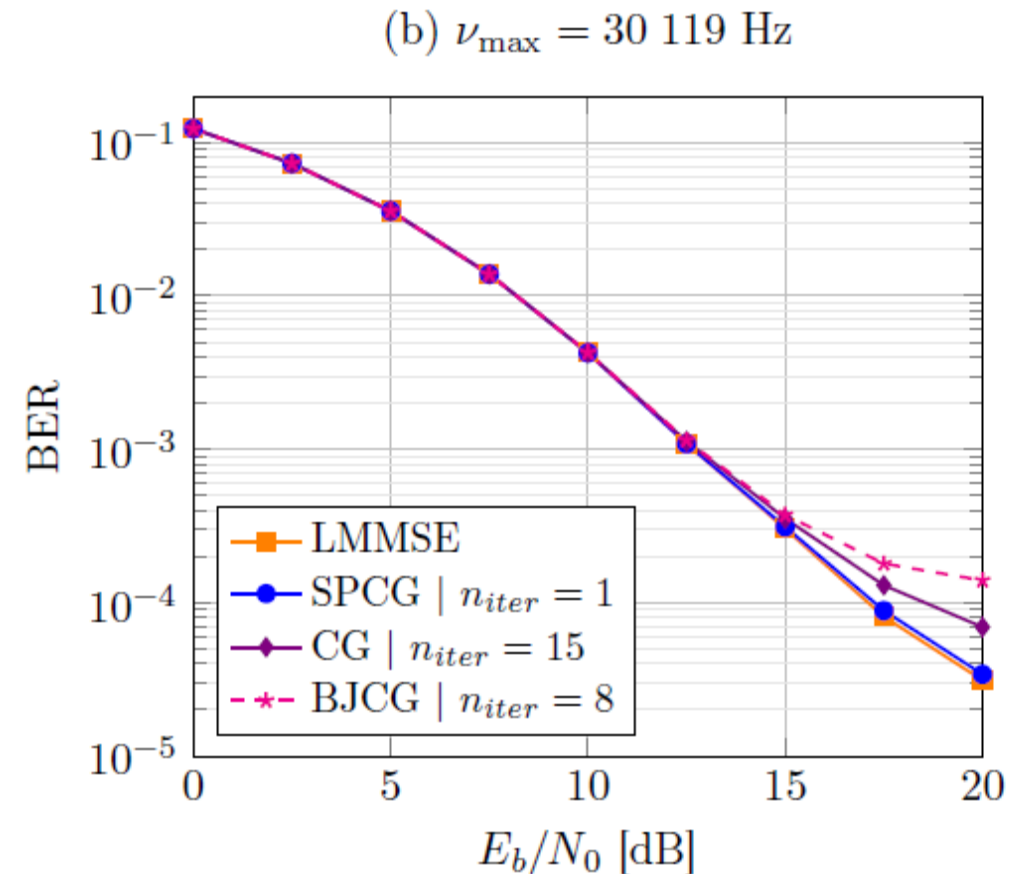
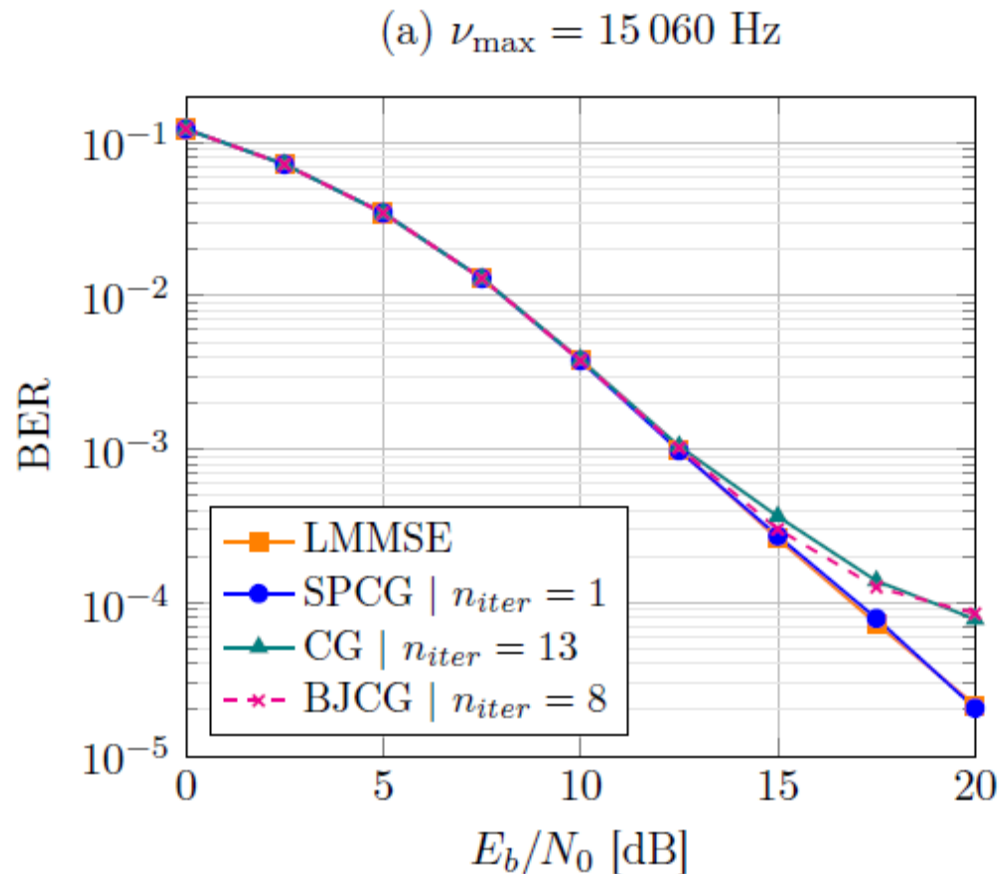
On (3GPP) NTN-TDL-A channels – Similar results on NTN-TDL-B (NLoS channels) – OTFS and ODDM are equivalent

Conjugate gradient Equalization on NTN channel

CG : Conjugate gradient, approximate LMMSE iteratively in delay-Doppler

SPCG : Sparse preconditioned CG, process in the time-domain (where the channel is sparse)

BJCG : Block-Jacobi CG, process in delay-Doppler domain with only the main block-diagonal matrices



Conjugate gradient Equalization on NTN channel



for uncoded BER = 10^{-3} ($v_{max} = 15$ kHz)
SPCG with 1 iteration / CG with 15 iterations / SNR = 12 dB

Complexity

$$\text{CG} : O(n_{ite} \times MN \times S)$$

$$\text{SPCG} : O(MN \times (\log N + \alpha(n_{ite} + \eta) + \beta P^2))$$

$$\text{MPA} : O(n_{ite} \times MN \times S \times |\mathcal{A}|)$$

$$\text{MRC} : O(n_{ite} \times MN \times L)$$

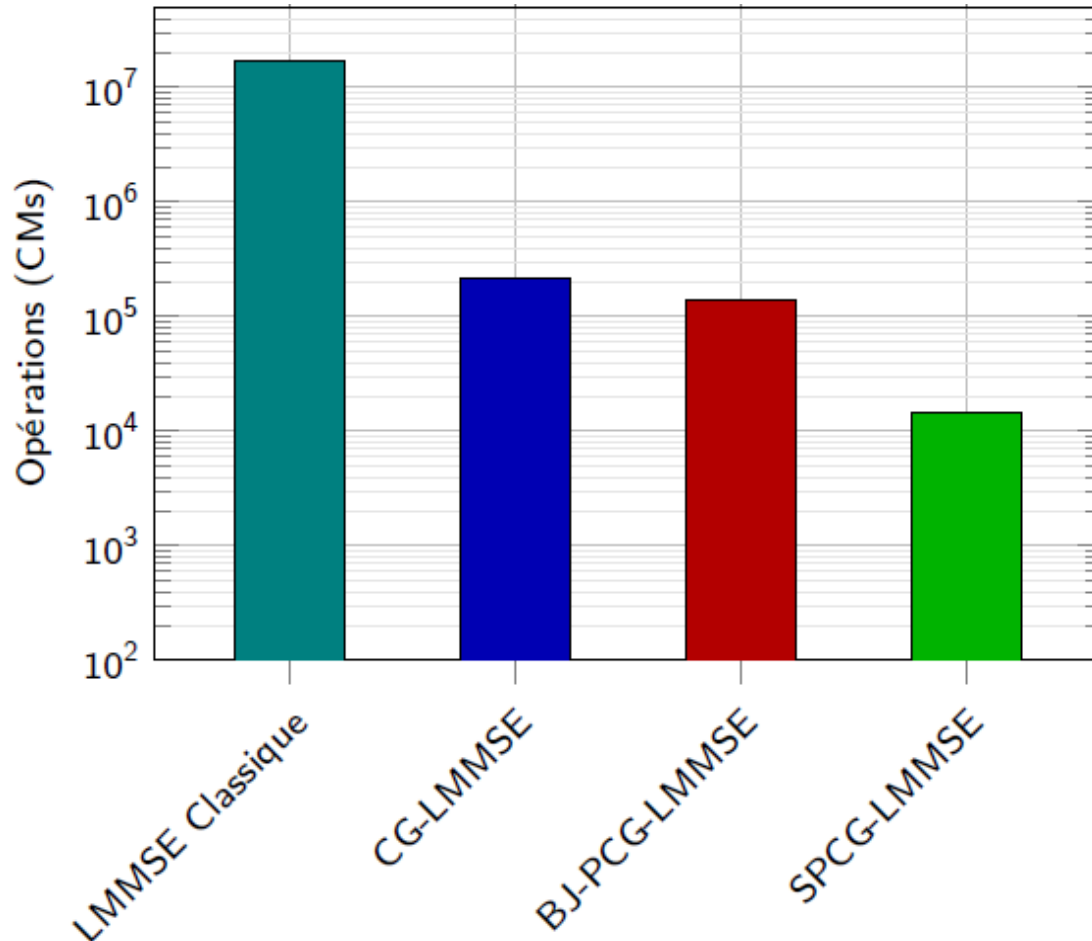
With

- $M = N = 16$
- $S = \alpha N$
- $\alpha = L + 1$ where L is the number of taps
- $L = 3$ taken from CP length (CPlen+1)
- $P = 4$ paths for NTN-B
- $\beta = K + 1$ with K number of Doppler bins (let $K=2$)
- $|\mathcal{A}| = 16$ (16QAM)

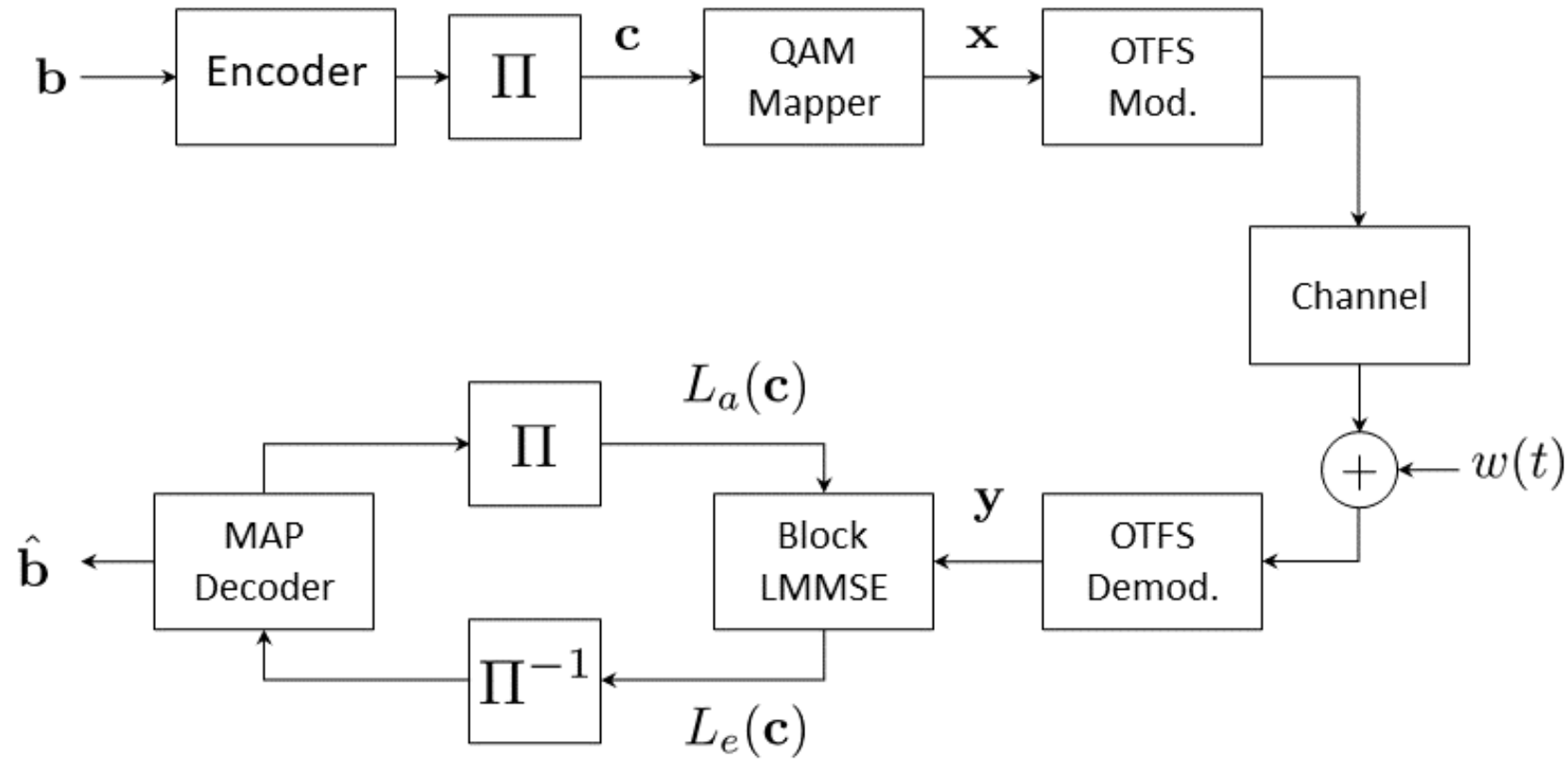
MPA = 983040 operations

MRC = 3840 (but doesn't work with small M and N !)

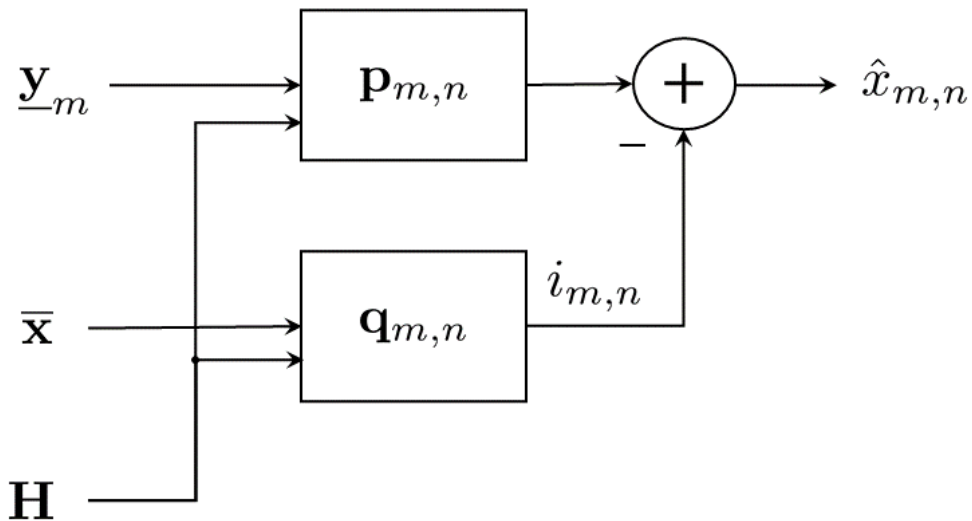
SPCG = 14022



Turbo Equalization in the Delay-Doppler Domain



Block LMMSE Equalizer



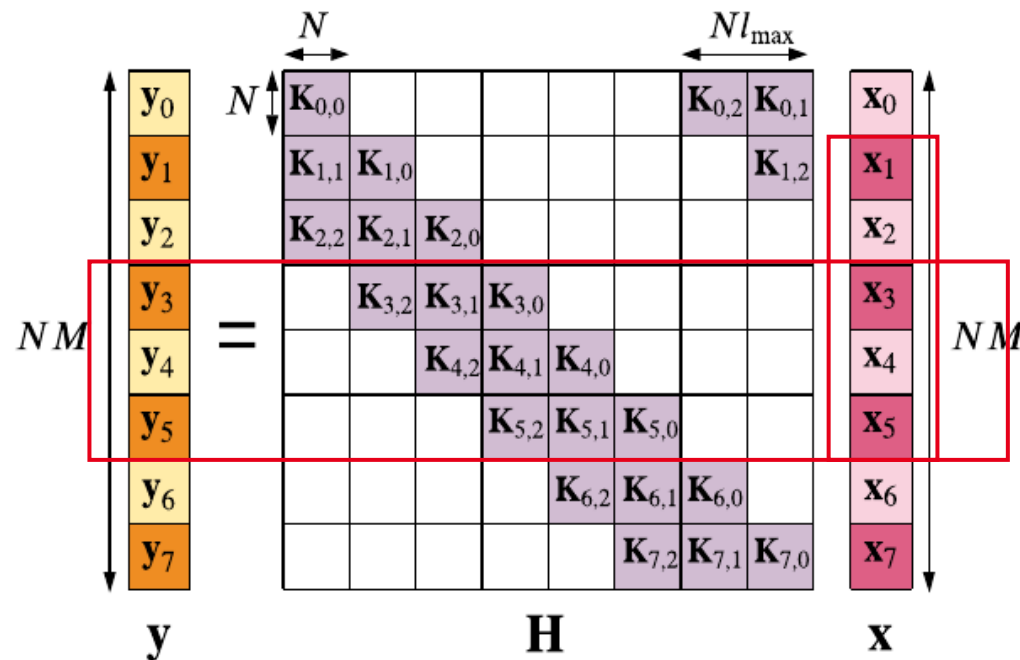
$$i_{m,n} = \mathbf{p}_{m,n}^H \underline{\mathbf{H}}_m \bar{\mathbf{x}}_{m \setminus n}$$

$$\mathbf{p}_{m,n} = \mathbf{R}_{m,n}^{-1} \mathbf{h}_{m,n}$$

$$\mathbf{R}_{m,n} = \sigma^2 \mathbf{I}_{LN} + \underline{\mathbf{H}}_m \mathbf{V}_{xx} \underline{\mathbf{H}}_m^H + (1 - \eta_{m,n}^2) \mathbf{h}_{m,n} \mathbf{h}_{m,n}^H$$

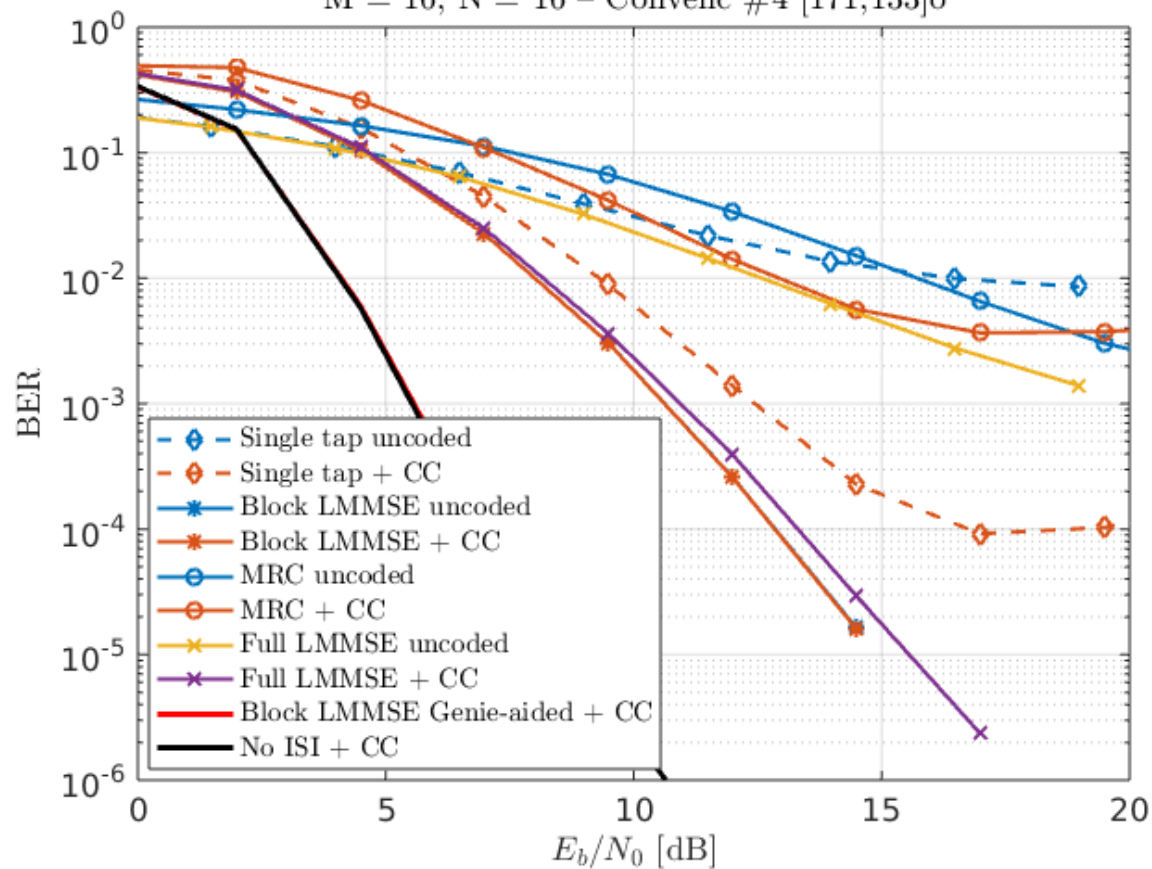
We consider L significant delays « taps » in the matrix $\mathbf{H}$

$$\begin{aligned} \underline{\mathbf{y}}_m &= [\mathbf{y}_m^T, \dots, \mathbf{y}_{m+L-1}^T]^T \\ &= \underline{\mathbf{H}}_m \underline{\mathbf{x}}_m + \underline{\mathbf{w}}_m \\ &= \mathbf{h}_{m,n} x_{m,n} + \underline{\mathbf{H}}_m \underline{\mathbf{x}}_{m \setminus n} + \underline{\mathbf{w}}_m \end{aligned}$$

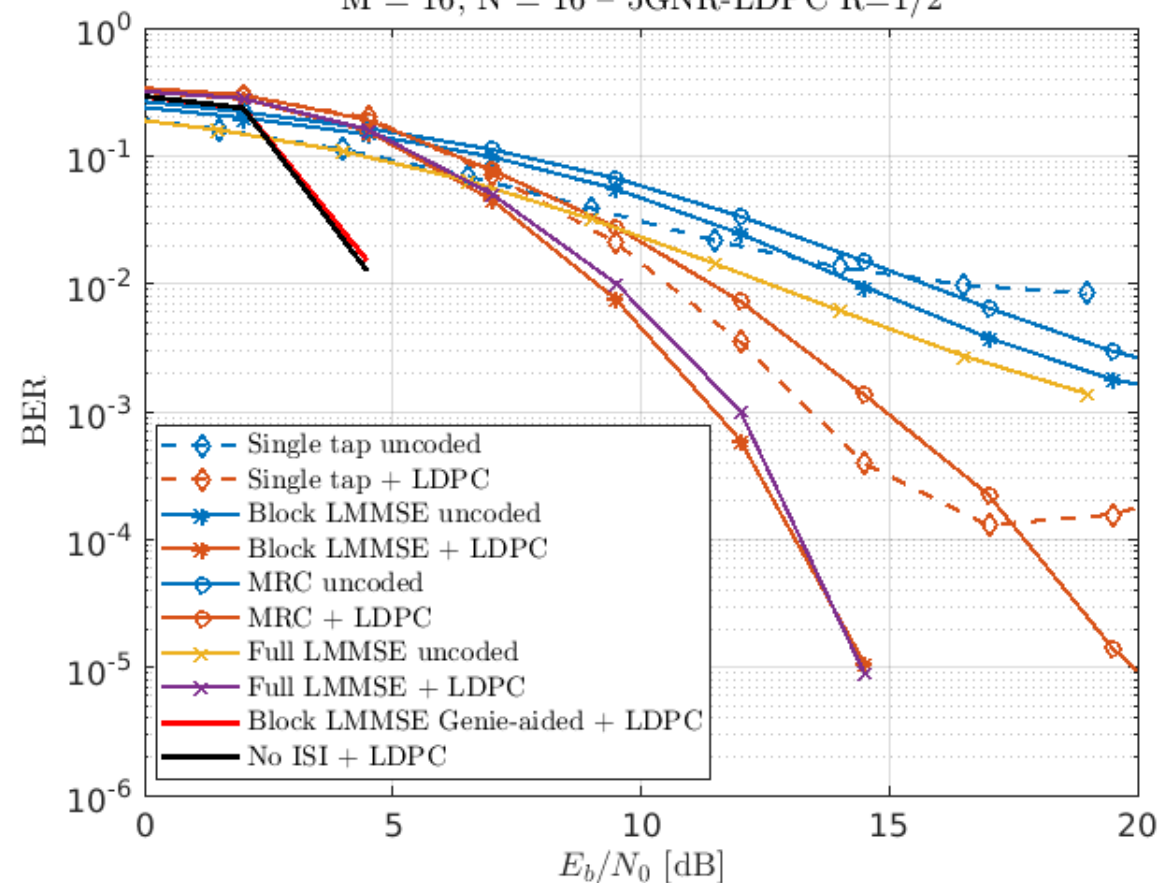


Block-LMMSE vs. MRC

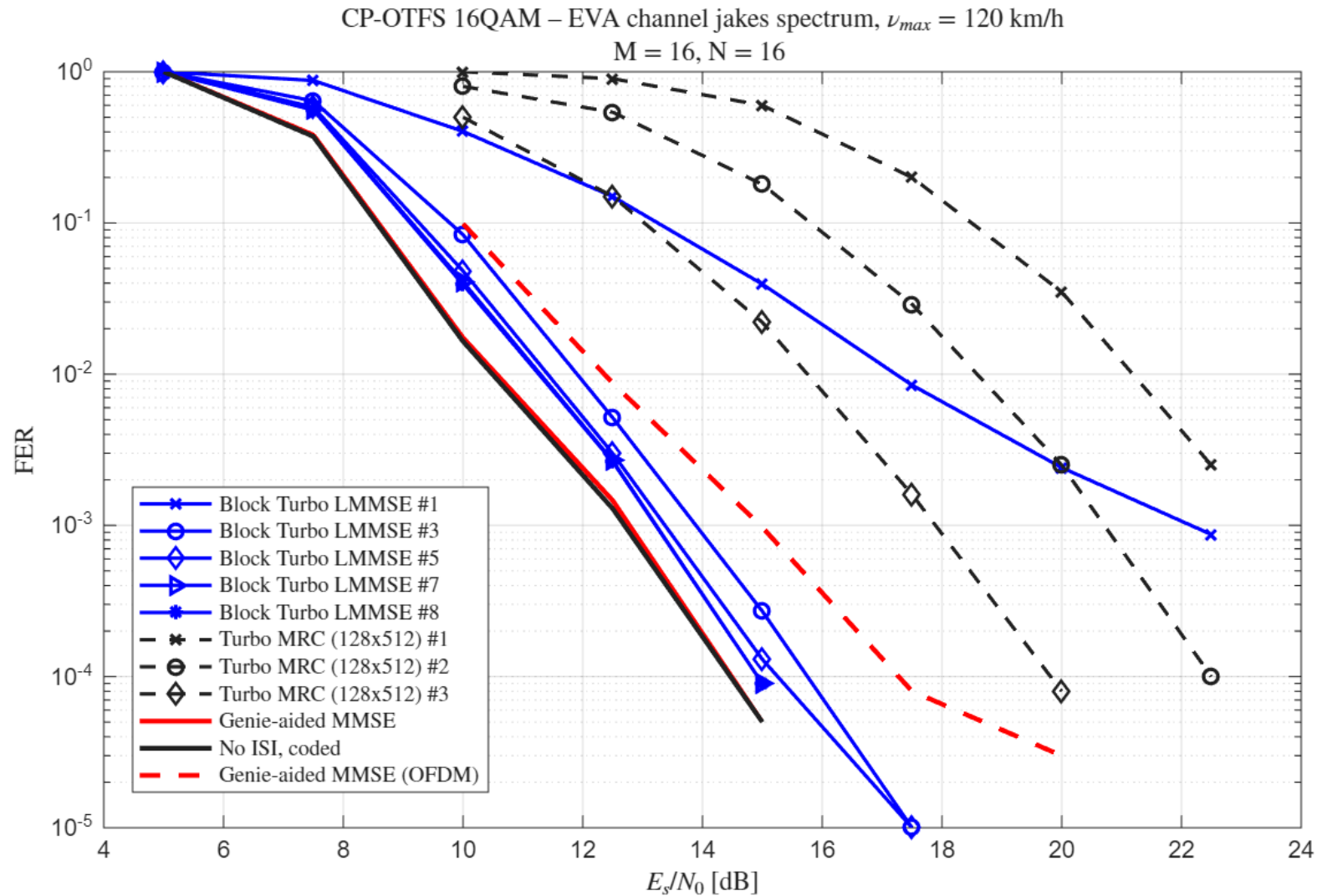
ZP-OTFS 16QAM – EVA channel jakes spectrum, $\nu_{max} = 120$ km/h
 $M = 16, N = 16$ – Convenc #4 [171,133]o



ZP-OTFS 16QAM – EVA channel jakes spectrum, $\nu_{max} = 120$ km/h
 $M = 16, N = 16$ – 5GNR-LDPC R=1/2



Turbo-LMMSE vs. Turbo-MRC



Block Turbo-LMMSE with
[171,133] convolutional encoder
Versus Turbo-MRC with 5GNR
LDPC

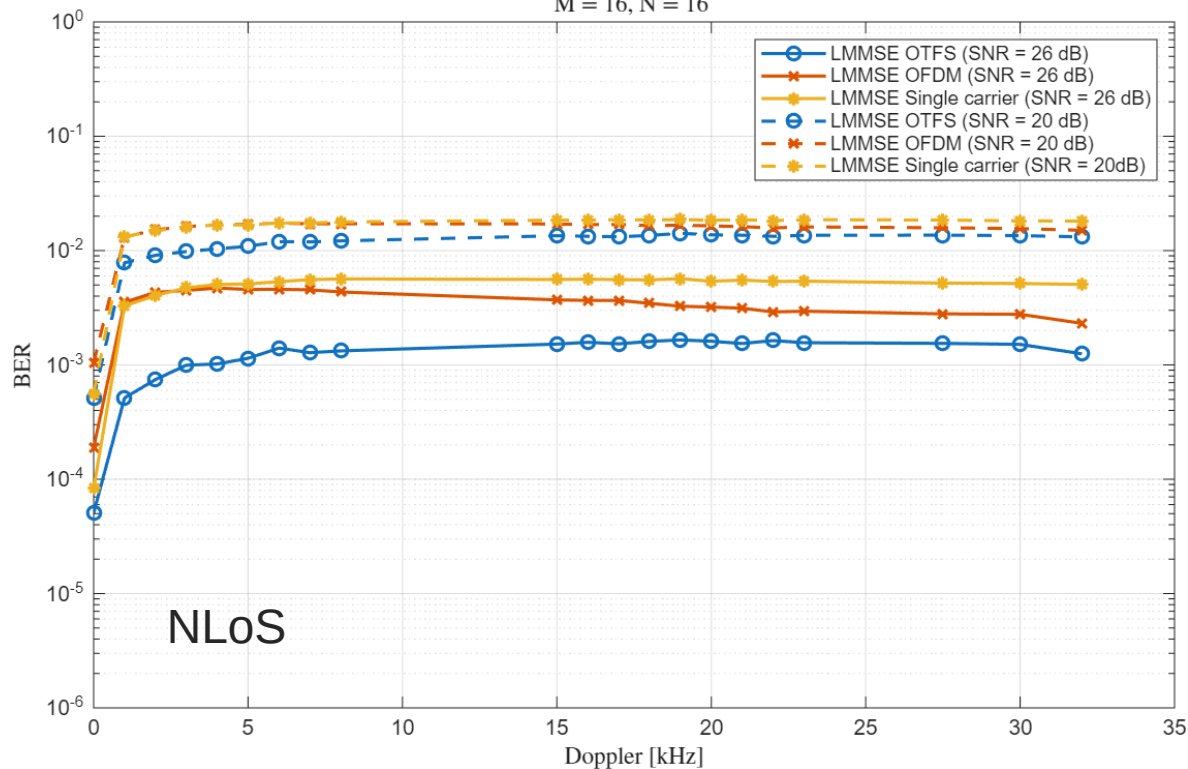
~ 5 dB SNR gain at the 3rd
iteration vs. Turbo-MRC in the
same channel

Better performance with less
computations because of
smaller block sizes

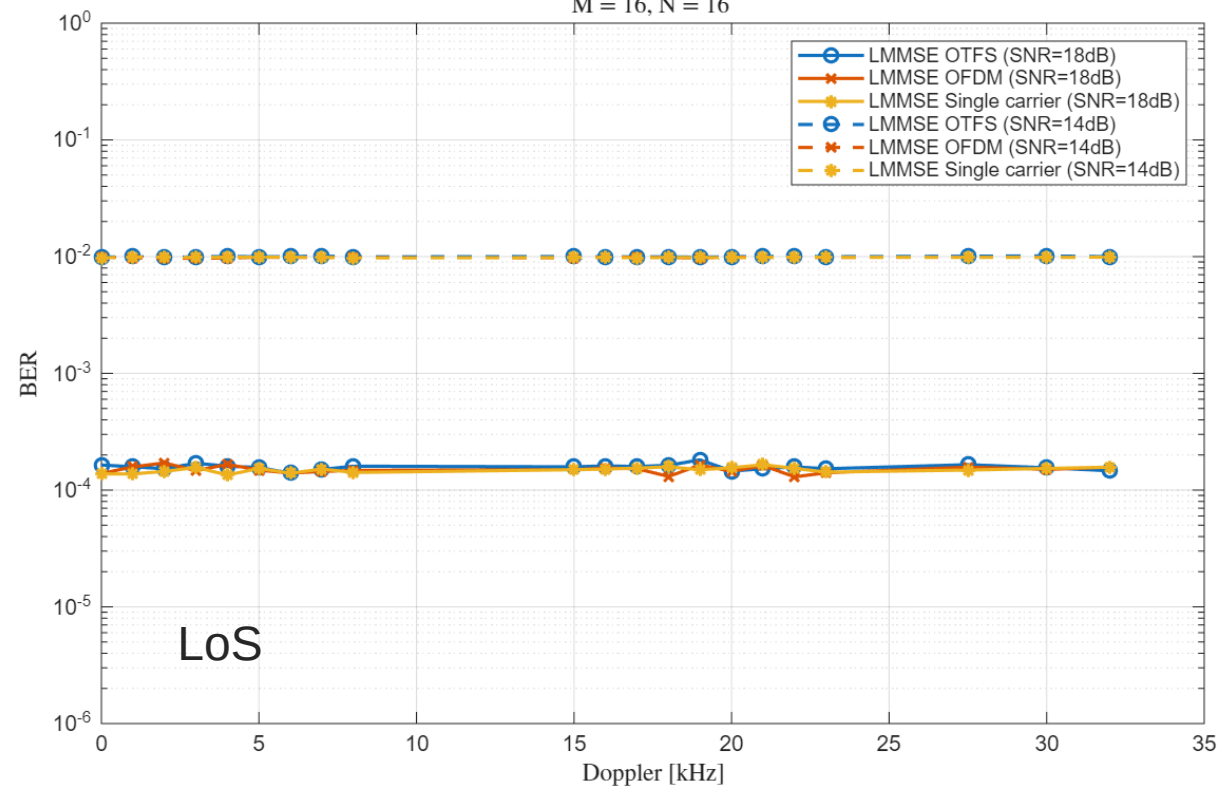
Performance is close to the
genie aided receiver

Doppler robustness ?

CP-OTFS 16QAM – EVA channel jakes spectrum,
 $M = 16, N = 16$



CP-OTFS 16QAM – E channel jakes spectrum
 $M = 16, N = 16$



The delay-Doppler waveforms are not « inherently robust to Doppler », but they rather offer a channel representation that can offer extra diversity which can be leveraged by an equalizer

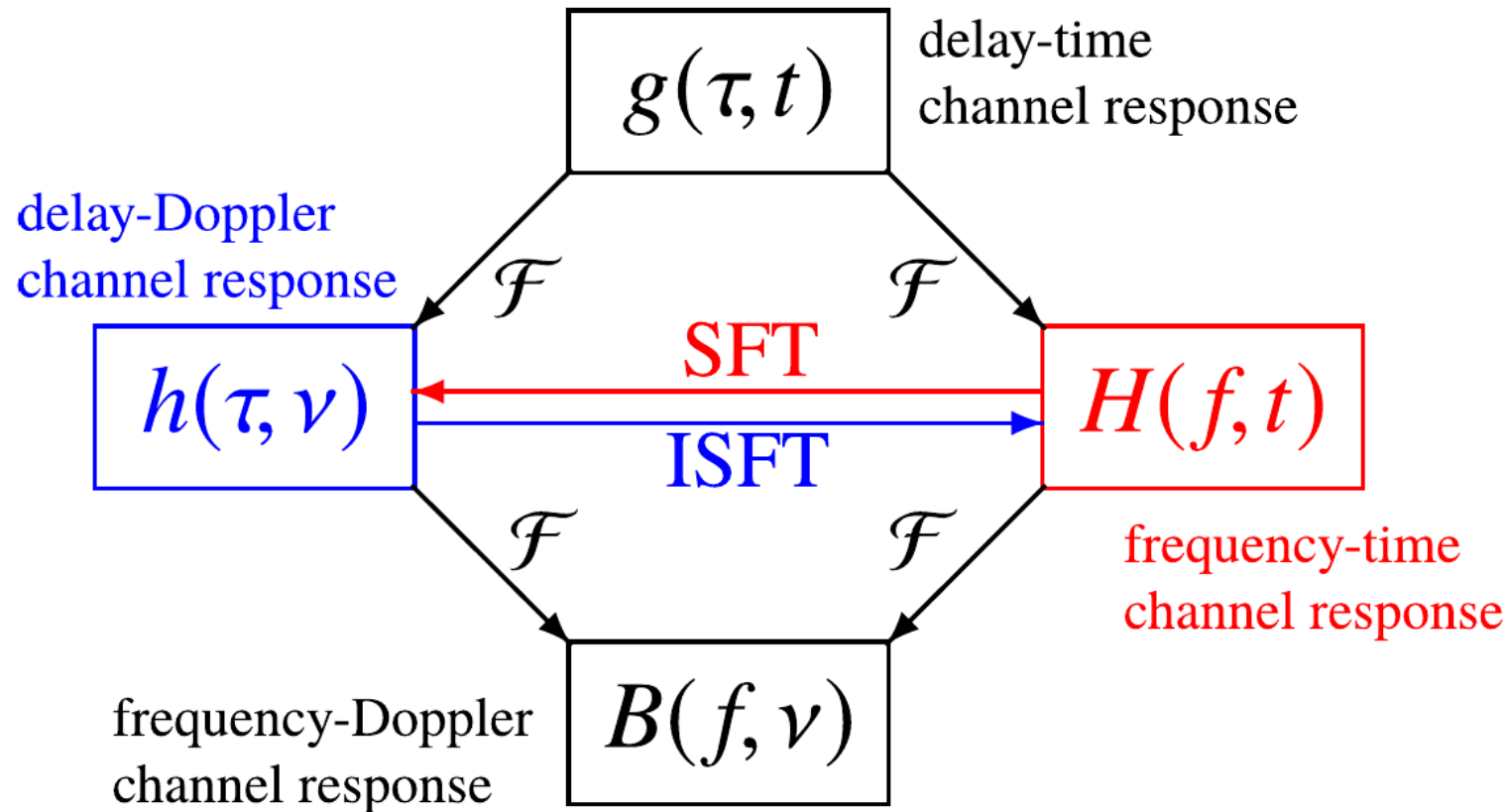
Conclusions

- Contributions:
 - Block turbo equalizer in the delay-Doppler domain
 - Performance evaluation in fractional delay-Doppler channels
 - Overall better performance than state of the art
 - Effective even on small delay-Doppler grid sizes (16×16)
 - Feedforward SPCG equalizers give good performance on NTN standardized channels with strong Dopplers → need to derive algorithms for turbo-equalization
- Future Work and perspective:
 - Question: does OTFS and ODDM truly provide additional diversity/robustness? (is the extra complexity worth it?)
 - Turbo equalization with conjugate gradient algorithms → see literature from massive MIMO
 - Nvidia AI Telco Engineer (July 2026) : automatic design of an equalizer for OTFS based on expectation-propagation to optimize the NMSE vs latency tradeoff



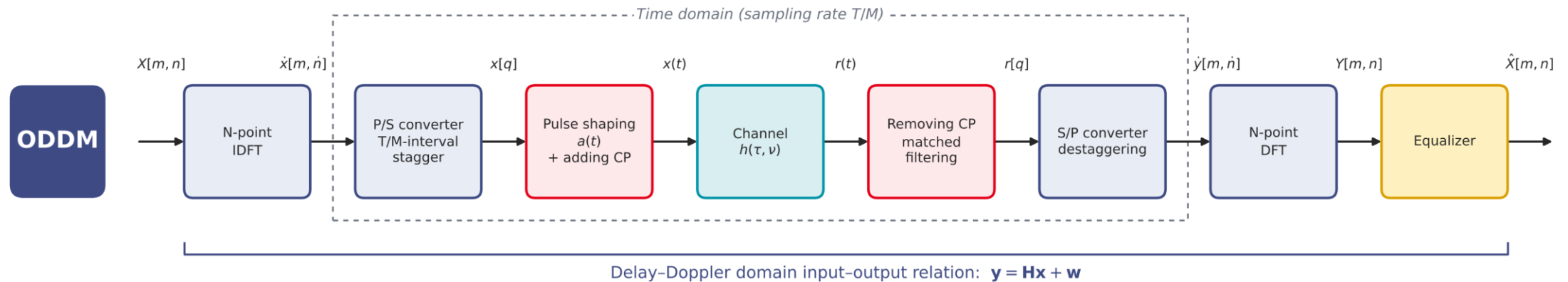
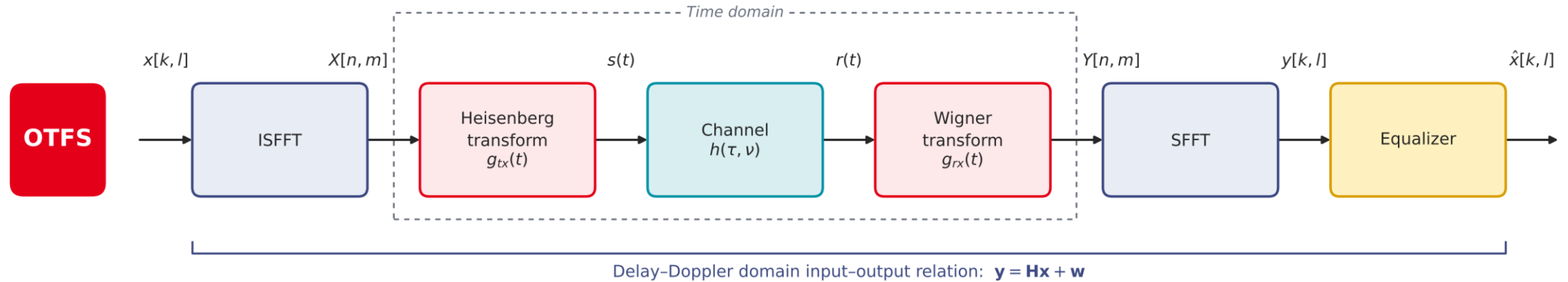
■ Thank you

Représentations du canal



Introduits par P. Bello, « Characterization of randomly time-variant linear channels », *IEEE Trans Comm*, 1963. Figure tirée de Hong *et al.* (2022) *op. cit.*

Delay-Doppler Input-Output Relations



Domain transform / conversion

Waveform shaping / matched filtering

Channel

Equalizer

Delay-Doppler Input-Output Relations



$\mathbf{r} = \mathbf{G} \cdot \mathbf{s} + \mathbf{w}$ $\longrightarrow$ Time-domain channel input-output relation with $\mathbf{G} \in \mathbb{C}^{NM \times NM}$

$$\mathbf{r}[q] = \sum_{l \in \mathcal{L}} g^s[l, q] \mathbf{s}[q - l] + \mathbf{w}[q], \quad q = 0, \dots, NM - 1$$

In time-frequency (OFDM), vectorized form:

With $\check{\mathbf{H}} = (\mathbf{I}_N \otimes \mathbf{F}_M) \cdot \mathbf{G} \cdot (\mathbf{I}_N \otimes \mathbf{F}_M^\dagger)$ et $\check{\mathbf{w}} = (\mathbf{I}_N \otimes \mathbf{F}_M) \cdot \mathbf{w}$.

$$\check{\mathbf{y}} = \check{\mathbf{H}} \cdot \check{\mathbf{x}} + \check{\mathbf{w}}$$

$$\check{\mathbf{x}} = (\mathbf{I}_N \otimes \mathbf{F}_M) \cdot \mathbf{s}$$

$$\check{\mathbf{y}} = (\mathbf{I}_N \otimes \mathbf{F}_M) \cdot \mathbf{r}$$

NOTE: The operation $(\mathbf{I}_N \otimes \mathbf{F}_M^H)$ is equivalent to stacking N OFDM symbols of M points, M being the FFT size

In delay-Doppler (OTFS/ODDM), vectorized form:

$$\mathbf{y} = \mathbf{H} \cdot \mathbf{x} + \mathbf{z}$$

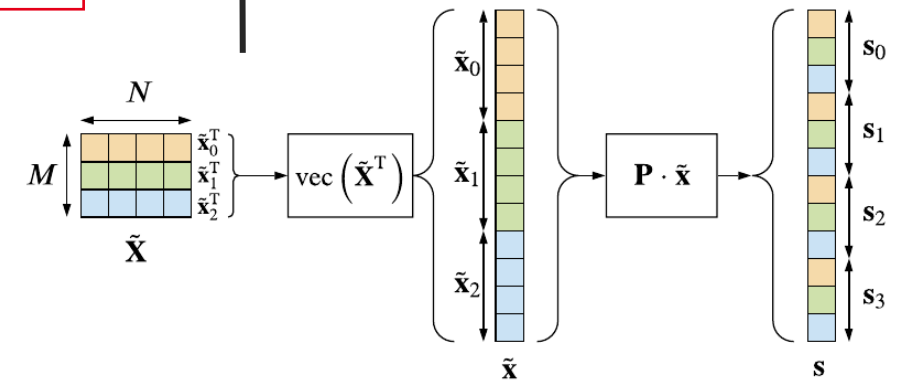
With $\mathbf{H} = (\mathbf{I}_M \otimes \mathbf{F}_N) \cdot (\mathbf{P}^T \cdot \mathbf{G} \cdot \mathbf{P}) \cdot (\mathbf{I}_M \otimes \mathbf{F}_N^\dagger) \in \mathbb{C}^{NM \times NM}$

$$\mathbf{z} = (\mathbf{I}_M \otimes \mathbf{F}_N) \cdot (\mathbf{P}^T \cdot \mathbf{w})$$

$$\mathbf{s} = \mathbf{P} \cdot (\mathbf{I}_M \otimes \mathbf{F}_N^\dagger) \cdot \mathbf{x}$$

$$\mathbf{r} = \mathbf{P} \cdot (\mathbf{I}_M \otimes \mathbf{F}_N^\dagger) \cdot \mathbf{y}$$

NOTE(BIS) :
 $(\mathbf{A} \otimes \mathbf{B})^{-1} = \mathbf{A}^{-1} \otimes \mathbf{B}^{-1}$



Detection Algorithms

Algorithm 1: MRC in delay-Doppler domain.

```

1 Input:  $\mathbf{H}$ ,  $\mathbf{D}_m$ ,  $\mathbf{y}_m$ ,  $\mathbf{x}_m = \mathbf{0}_N \quad \forall m = 0, \dots, M - 1$ 
2 for  $i=1:\text{max iterations}$  do
3   for  $m = 0 : M' - 1$  do
4     for  $l \in \mathcal{L}$  do
5        $\mathbf{b}_m^l = \mathbf{y}_{m+l} - \sum_{p \neq l} \mathbf{K}_{m+l,p} \cdot \hat{\mathbf{x}}_{m+l-p}$ 
6     end
7      $\mathbf{g}_m = \sum_{l \in \mathcal{L}} \mathbf{K}_{m+l,l}^\dagger \cdot \mathbf{b}_m^l$ 
8      $\mathbf{c}_m = \mathbf{D}_m^{-1} \cdot \mathbf{g}_m$ 
9      $\hat{\mathbf{x}}_m = \mathcal{D}(\mathbf{c}_m)$  (or  $\hat{\mathbf{x}}_m = \mathbf{c}_m$ )5
10  end
11 end
12 Output:  $\hat{\mathbf{x}}_m$ 

```

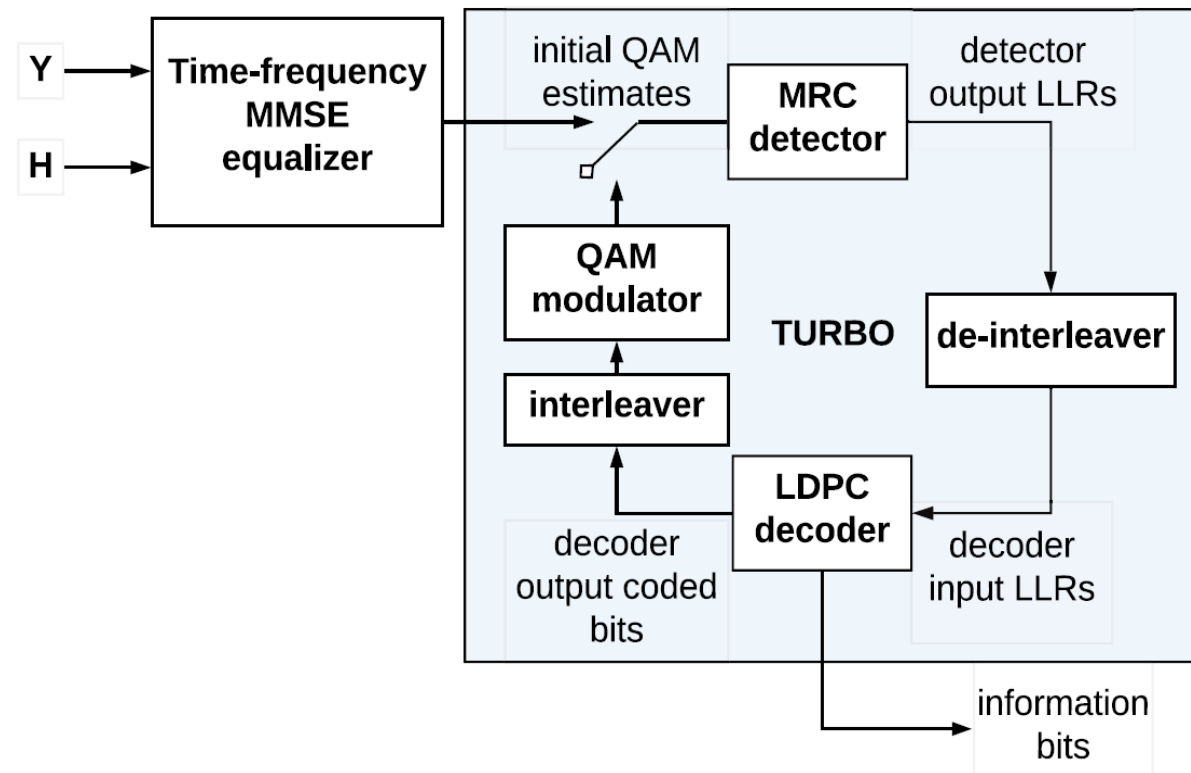
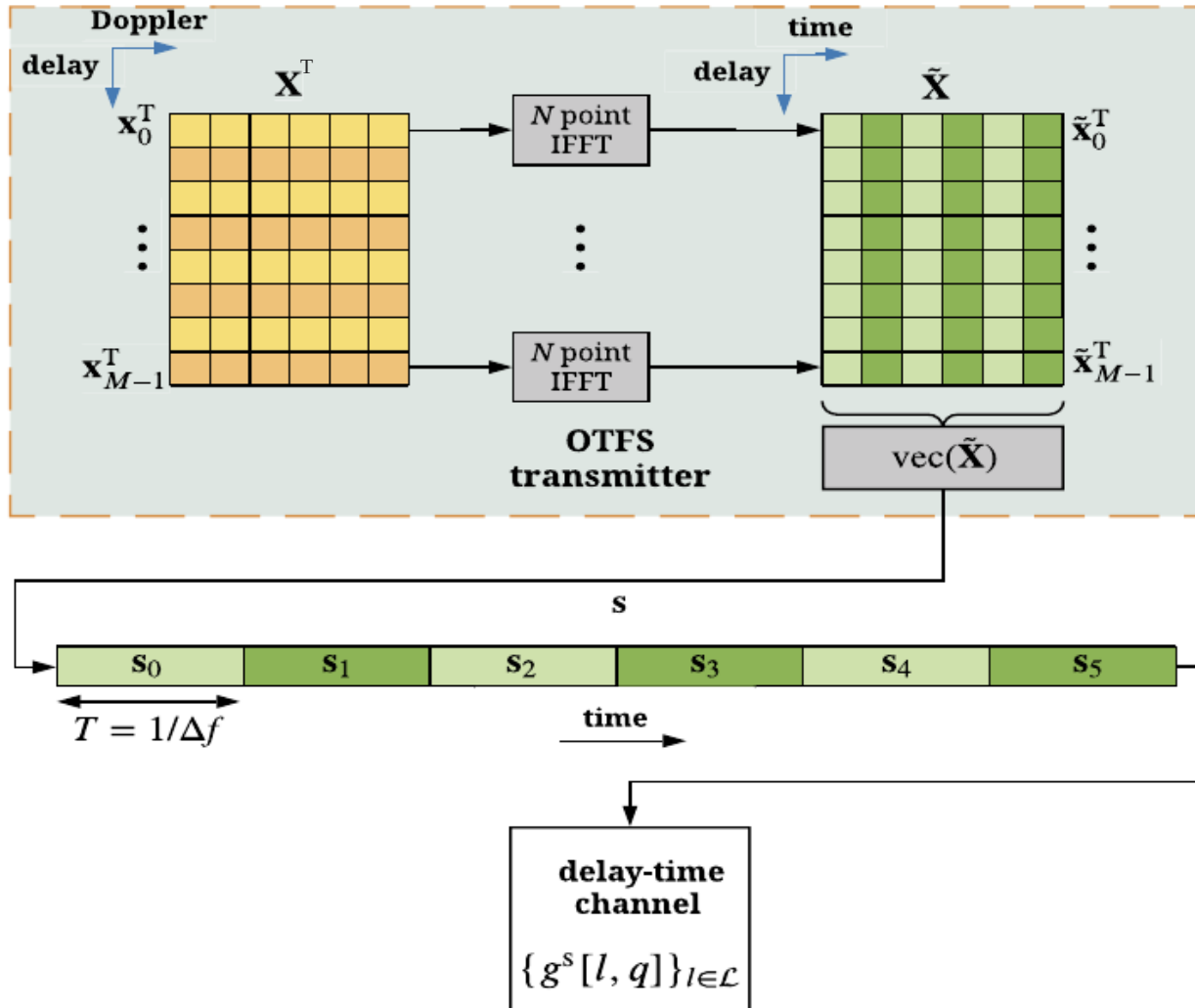


Fig. 8. OTFS iterative rake turbo decoder operation.

From Hong, Thaj, Viterbo, *Delay-Doppler Communications*, Elsevier, 2022

and Thaj & Viterbo, *IEEE, Trans Veh. Tech* 2020

Relations entrées-sorties en retard-Doppler



- retard-Doppler vers fréquence-temps :

$$\mathbf{X}_{\text{tf}} = \mathbf{F}_M \cdot \mathbf{X} \cdot \mathbf{F}_N^\dagger$$

- Fréquence-temps vers retard-temps (OFDM)

$$\tilde{\mathbf{X}} = \mathbf{F}_M^\dagger \cdot \mathbf{X}_{\text{tf}} = \underbrace{\mathbf{F}_M^\dagger \cdot \mathbf{F}_M}_{\mathbf{I}_M} \cdot \mathbf{X} \cdot \mathbf{F}_N^\dagger = \mathbf{X} \cdot \mathbf{F}_N^\dagger.$$

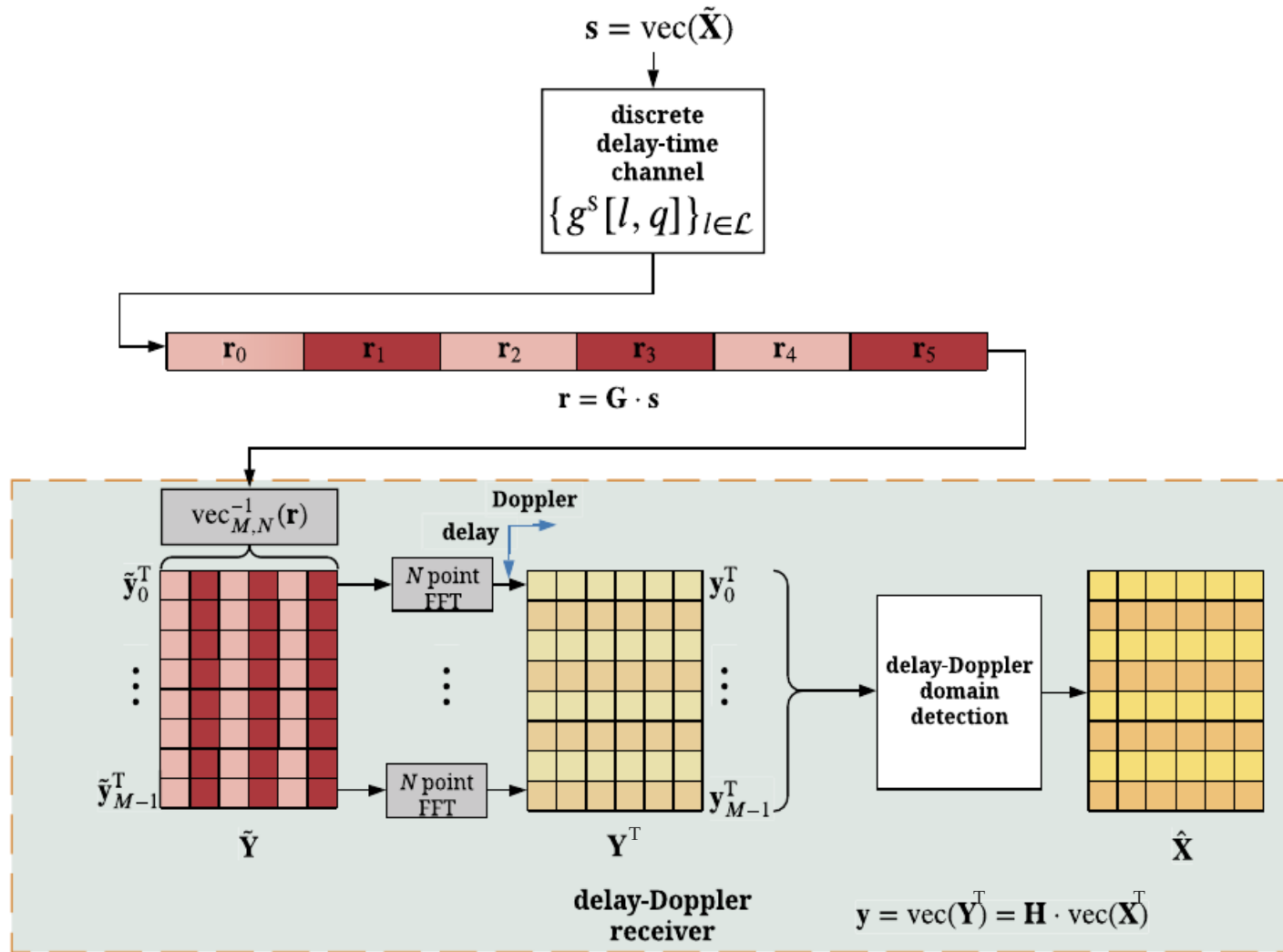
$$\mathbf{s} = \text{vec}(\tilde{\mathbf{X}}) = \text{vec}(\mathbf{X} \cdot \mathbf{F}_N^\dagger)$$

- Sous forme vectorisée pour les entrées ($\mathbf{X} \rightarrow \mathbf{x}$)

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_0 \\ \vdots \\ \mathbf{x}_{M-1} \end{bmatrix} = \text{vec}(\mathbf{X}^T)$$

$$\mathbf{s} = \mathbf{P} \cdot (\mathbf{I}_M \otimes \mathbf{F}_N^\dagger) \cdot \mathbf{x}$$

Relations entrées-sorties en retard-Doppler



- Temps vers fréquence-temps (OFDM) :

$$\tilde{\mathbf{Y}} = \text{vec}^{-1}_{M,N}(\mathbf{r}) \text{ or } \mathbf{r} = \text{vec}(\tilde{\mathbf{Y}})$$

$$\mathbf{Y}_{\text{tf}} = \mathbf{F}_M \cdot \tilde{\mathbf{Y}}.$$

- Retour dans le domaine retard-Doppler

$$\mathbf{Y} = \mathbf{F}_M^\dagger \cdot \mathbf{Y}_{\text{tf}} \cdot \mathbf{F}_N = \tilde{\mathbf{Y}} \cdot \mathbf{F}_N$$

- Sous forme vectorisée :

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_0 \\ \vdots \\ \mathbf{y}_{M-1} \end{bmatrix} = \text{vec}(\mathbf{Y}^T)$$

$$\mathbf{r} = \mathbf{P} \cdot (\mathbf{I}_M \otimes \mathbf{F}_N^\dagger) \cdot \mathbf{y}.$$

Canal équivalent discret en bande de base

$$r(t) = \iint h(\tau, \nu) s(t - \tau) e^{j2\pi\nu(t-\tau)} d\tau d\nu$$
$$= \int g(\tau, t) s(t - \tau) d\tau,$$

Où $h(\tau, \nu) = \mathcal{F}_t\{g(\tau, t)\}$ est la fonction d'étalement du canal

$$r[q] = \sum_l g^s[l, q] s[q - l], \quad q = 0, \dots, NM - 1,$$

Après discrétisation à $t = qT_s = q \frac{T}{M} = \frac{q}{M\Delta f}$.
En retard le canal est discrétisé selon $\tau = l \frac{T}{M}$ pour $l = 0, \dots, M - 1$

Canal discret en bande de base :

$$g^s[l, q] = \sum_{\ell \in \mathcal{L}} \left(\sum_{\kappa \in \mathcal{K}_\ell} v_\ell(\kappa) z^{\kappa(q-l)} \right) \text{sinc}(l - \ell) \quad \text{où } z = e^{\frac{j2\pi}{NM}}$$

$$v_\ell(\kappa) = \begin{cases} g_i, & \text{if } \ell = \ell_i = \tau_i / M \Delta f \text{ and } \kappa = \kappa_i = \nu_i / NT \\ 0, & \text{otherwise.} \end{cases}$$

« réponse en Doppler » sur la grille

$\mathcal{L}$ est l'ensemble des retards discrets normalisés du canal sur la grille et $\mathcal{K}_l$ est l'ensemble des Dopplers discrets normalisés du canal pour les trajets partageant le même $l^{\text{ième}}$ retard normalisé.

(Pas forcément entiers)

- Canal multi-trajets mobile

$$r(t) = \iint h(\tau, \nu) s(t - \tau) e^{j2\pi\nu(t-\tau)} d\tau d\nu \quad \text{Où } h(\tau, \nu) = \mathcal{F}_t\{g(\tau, t)\} \text{ est la fonction d'étalement du canal}$$

$$= \int g(\tau, t) s(t - \tau) d\tau,$$

$$r[q] = \sum_l g^s[l, q] s[q - l], \quad q = 0, \dots, NM - 1,$$

Après discrétisation à $t = qT_s = q \frac{T}{M} = \frac{q}{M\Delta f}$.
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Canal discret en bande de base :

$$g^s[l, q] = \sum_{\ell \in \mathcal{L}} \left(\sum_{\kappa \in \mathcal{K}_\ell} v_\ell(\kappa) z^{\kappa(q-l)} \right) \text{sinc}(l - \ell) \quad \text{où } z = e^{\frac{j2\pi}{NM}}$$

$$v_\ell(\kappa) = \begin{cases} g_i, & \text{if } \ell = \ell_i = \tau_i / M\Delta f \text{ and } \kappa = \kappa_i = \nu_i / NT \\ 0, & \text{otherwise.} \end{cases} \quad \text{« réponse en Doppler » sur la grille}$$

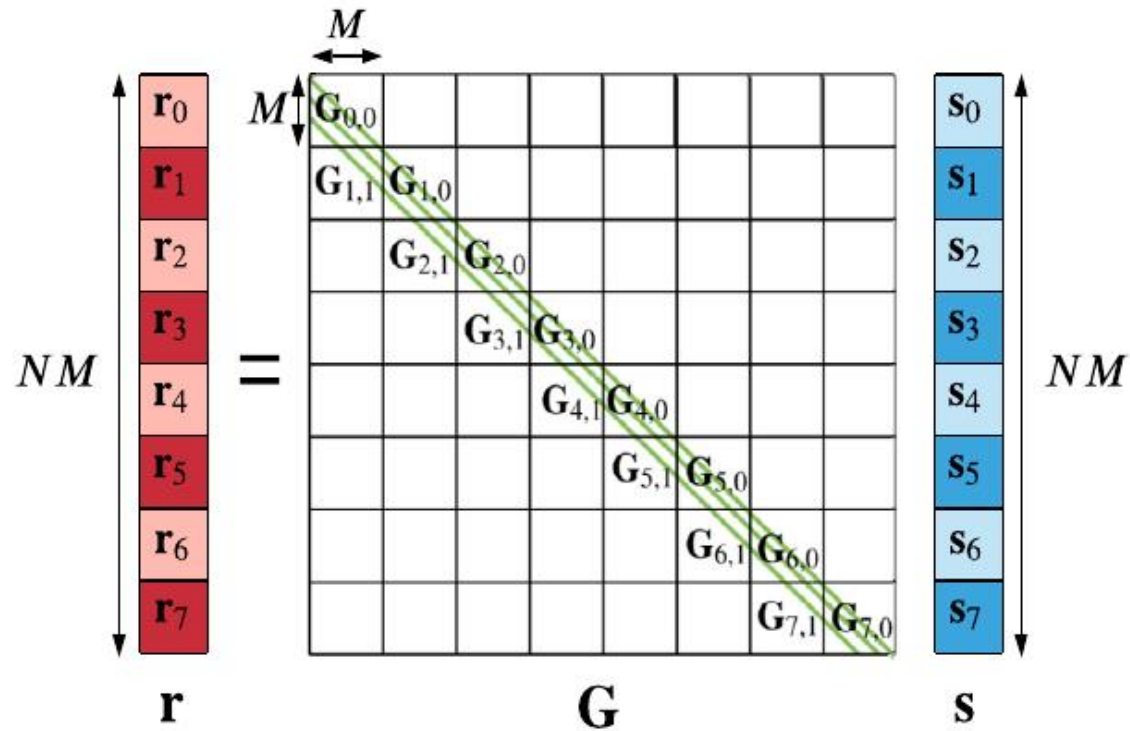
$\mathcal{L}$ est l'ensemble des retards discrets normalisés du canal sur la grille et $\mathcal{K}_\ell$ est l'ensemble des Dopplers discrets normalisés du canal pour les trajets partageant le même $l^{\text{ième}}$ retard normalisé. (*Pas forcément entiers*)

- Canal multi-trajets mobile

Hypothèse : le canal se résume à trois types de paramètres $(g_i, \tau_i, \nu_i)_{i=1}^P \rightarrow$ valeur du coeff. pour un retard et un doppler

1.
$$h(\tau, \nu) = \sum_{i=1}^P g_i \delta(\tau - \tau_i) \delta(\nu - \nu_i)$$
 Fonction d'étalement
2.
$$h(\tau, \nu) = \sum_{\ell \in \mathcal{L}} \sum_{\kappa \in \mathcal{K}_\ell} v_\ell(\kappa) \delta(\tau - \ell T/M) \delta(\nu - \kappa \Delta f/N), \quad \text{où} \quad v_\ell(\kappa) = \begin{cases} g_i, & \text{if } \ell = \ell_i \text{ and } \kappa = \kappa_i \\ 0, & \text{otherwise} \end{cases}$$
3.
$$g(\tau, t) = \int_{\nu} h(\tau, \nu) e^{j2\pi \nu(t-\tau)} d\nu$$
 Réponse impulsionnelle
4.
$$g(\tau, t) = \sum_{\ell \in \mathcal{L}} \sum_{\kappa \in \mathcal{K}_\ell} v_\ell(\kappa) e^{j2\pi \kappa \frac{\Delta f}{N}(t-\ell T/M)} \delta(\tau - \ell T/M)$$
 en injectant (2) dans (3)
5.
$$g^s[l, q] = \sum_{\ell \in \mathcal{L}} \left(\sum_{\kappa \in \mathcal{K}_\ell} v_\ell(\kappa) z^{\kappa(q-l)} \right) \text{sinc}(l - \ell)$$
 en discrétisant

Structures des matrices de canal

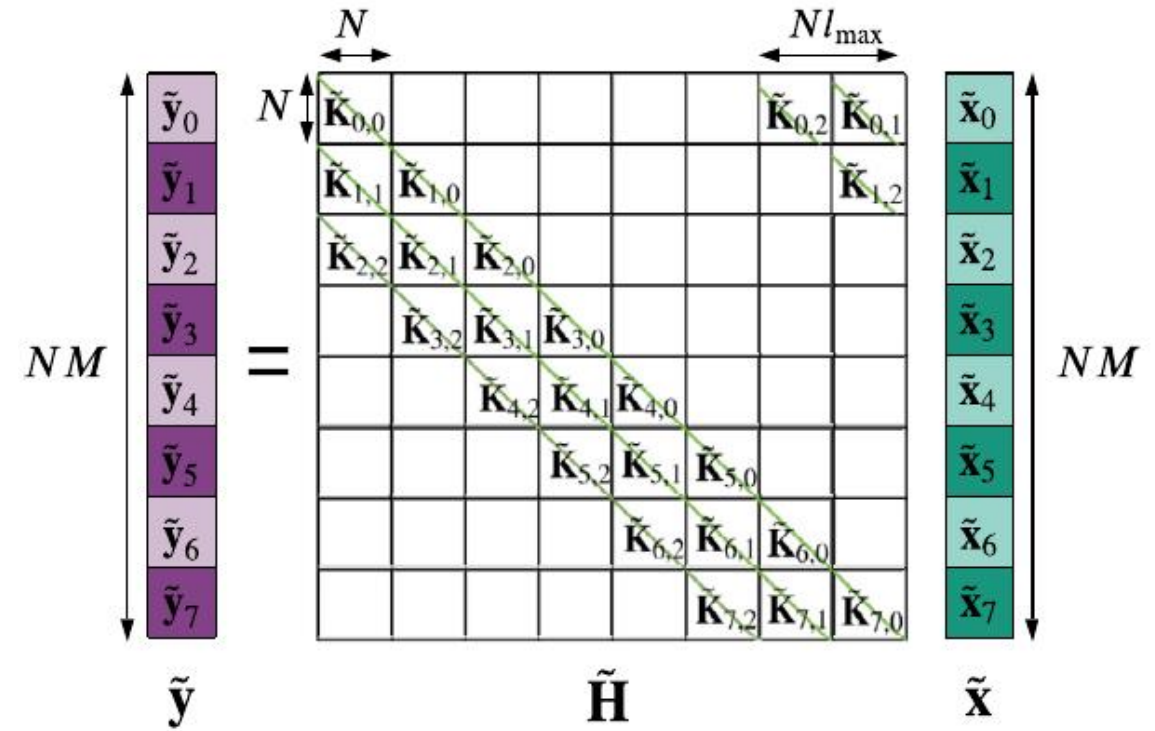


Relations dans le domaine temporel :

$G_{n,0}$ représente le canal traversé par le symbole n

$G_{n,1}$ représente l'interférence entre les symboles n et $n - 1$ (interférence entre blocs)

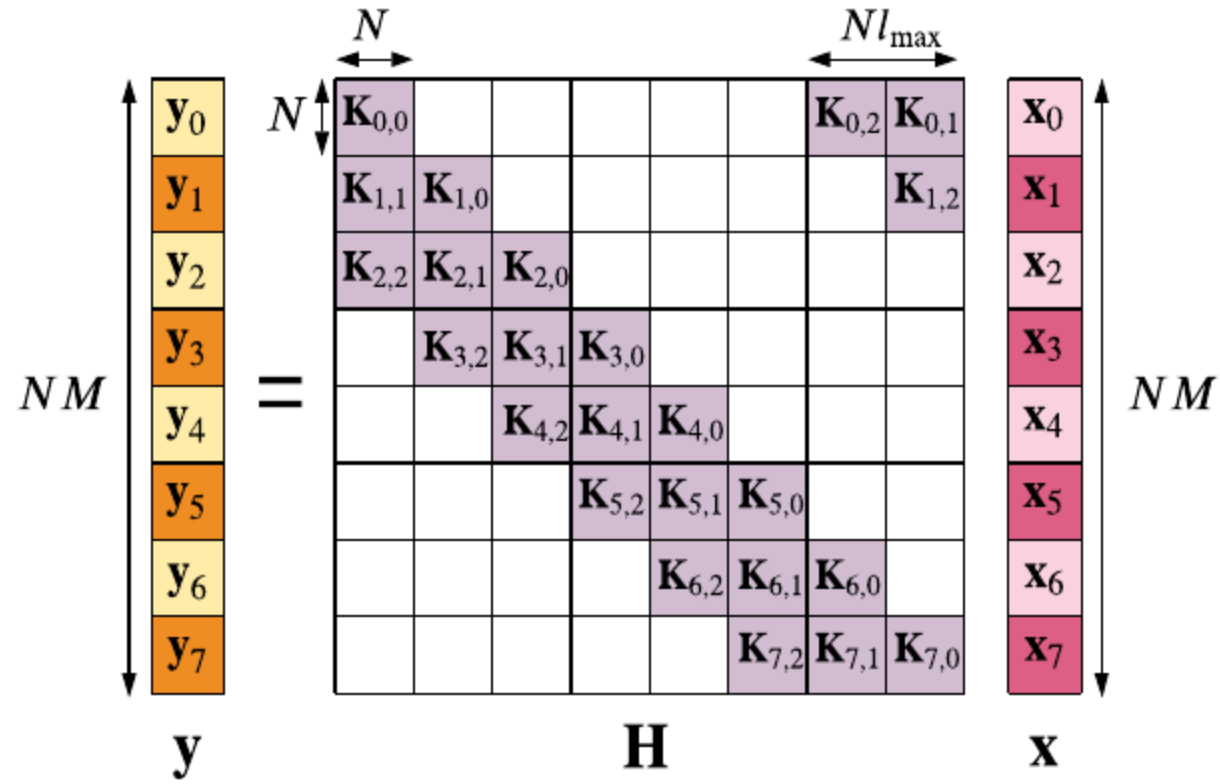
$$\mathbf{G}[q, q - l] = g^s[l, q] \quad \text{for } l \leq q \leq NM - 1$$



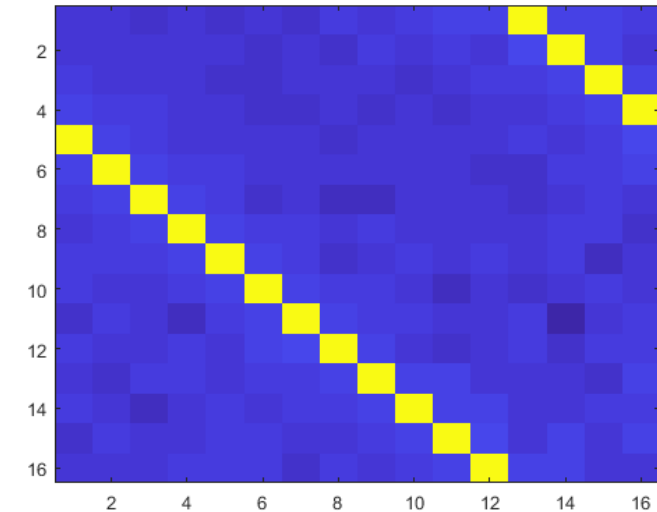
Relations dans le domaine retard-temps :

Les matrices $\tilde{K}_{n,m}$ sont diagonales et représentent les trajets de $\mathbf{G}$. Les termes hors diagonale de $\tilde{\mathbf{H}}$ proviennent de l'interférence entre blocs

Structure des matrices de canal



- Les termes d'interférence inter-blocs $\mathbf{K}(m,l)$ pour $l > m$ peuvent être éliminés par l'insertion d'intervalles de garde (zéro-padding)
- La matrice $\mathbf{H}$ est (en théorie) parcimonieuse



Dans le domaine retard-Doppler :

$$y_m = \sum_l \mathbf{K}_{m,l} \mathbf{x}_{m-l} + \mathbf{z}_m$$

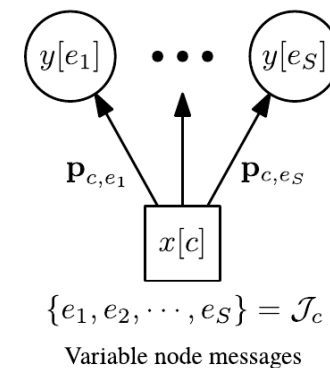
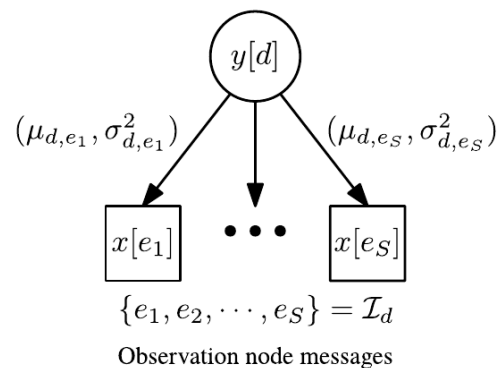
Exemple reproduit avec Matlab sur une grille 16x16 avec $\delta_D = 1$ kHz et un décalage Doppler de 4 kHz sur canal AWGN montrant le bloc $\mathbf{K}_{0,0}$.

- Algorithmes de Détection

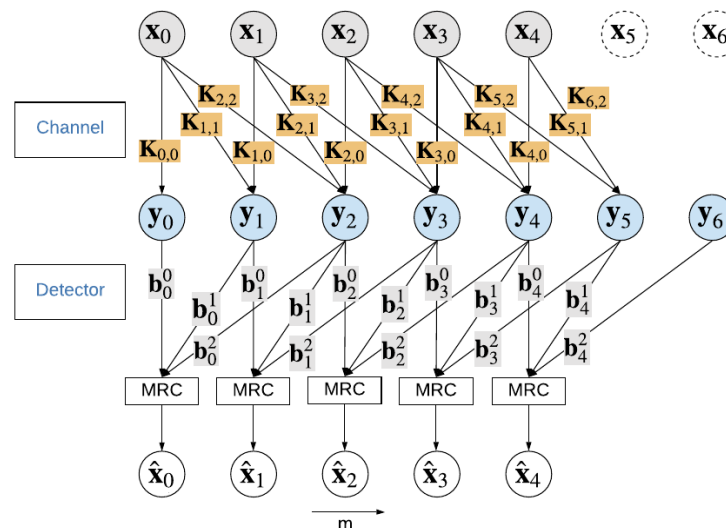
En délai-Doppler (OTFS), on a en version vectorisée : $\mathbf{y} = \mathbf{H} \cdot \mathbf{x} + \mathbf{z}$,

Avec $\mathbf{H} = (\mathbf{I}_M \otimes \mathbf{F}_N) \cdot (\mathbf{P}^T \cdot \mathbf{G} \cdot \mathbf{P}) \cdot (\mathbf{I}_M \otimes \mathbf{F}_N^\dagger) \in \mathbb{C}^{NM \times NM}$

Message Passing Algorithm (MPA)



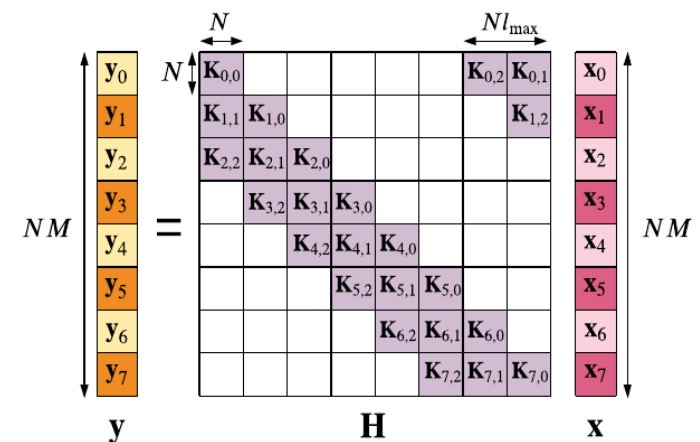
Maximal Ratio Combiner



LMMSE :

$$\hat{\mathbf{x}} = (\mathbf{H}^\dagger \cdot \mathbf{H} + \sigma_w^2 \mathbf{I}_{MN})^{-1} \cdot \mathbf{H}^\dagger \cdot \mathbf{y}.$$

Complexité : inversion de matrice de taille MN



- Algorithmes de Détection

Algorithm 1: MRC in delay-Doppler domain.

```

1 Input:  $\mathbf{H}$ ,  $\mathbf{D}_m$ ,  $\mathbf{y}_m$ ,  $\mathbf{x}_m = \mathbf{0}_N \quad \forall m = 0, \dots, M - 1$ 
2 for  $i=1:\text{max iterations}$  do
3   for  $m = 0 : M' - 1$  do
4     for  $l \in \mathcal{L}$  do
5        $\mathbf{b}_m^l = \mathbf{y}_{m+l} - \sum_{p \neq l} \mathbf{K}_{m+l,p} \cdot \hat{\mathbf{x}}_{m+l-p}$ 
6     end
7      $\mathbf{g}_m = \sum_{l \in \mathcal{L}} \mathbf{K}_{m+l,l}^\dagger \cdot \mathbf{b}_m^l$ 
8      $\mathbf{c}_m = \mathbf{D}_m^{-1} \cdot \mathbf{g}_m$ 
9      $\hat{\mathbf{x}}_m = \mathcal{D}(\mathbf{c}_m)$  (or  $\hat{\mathbf{x}}_m = \mathbf{c}_m$ )5
10  end
11 end
12 Output:  $\hat{\mathbf{x}}_m$ 

```

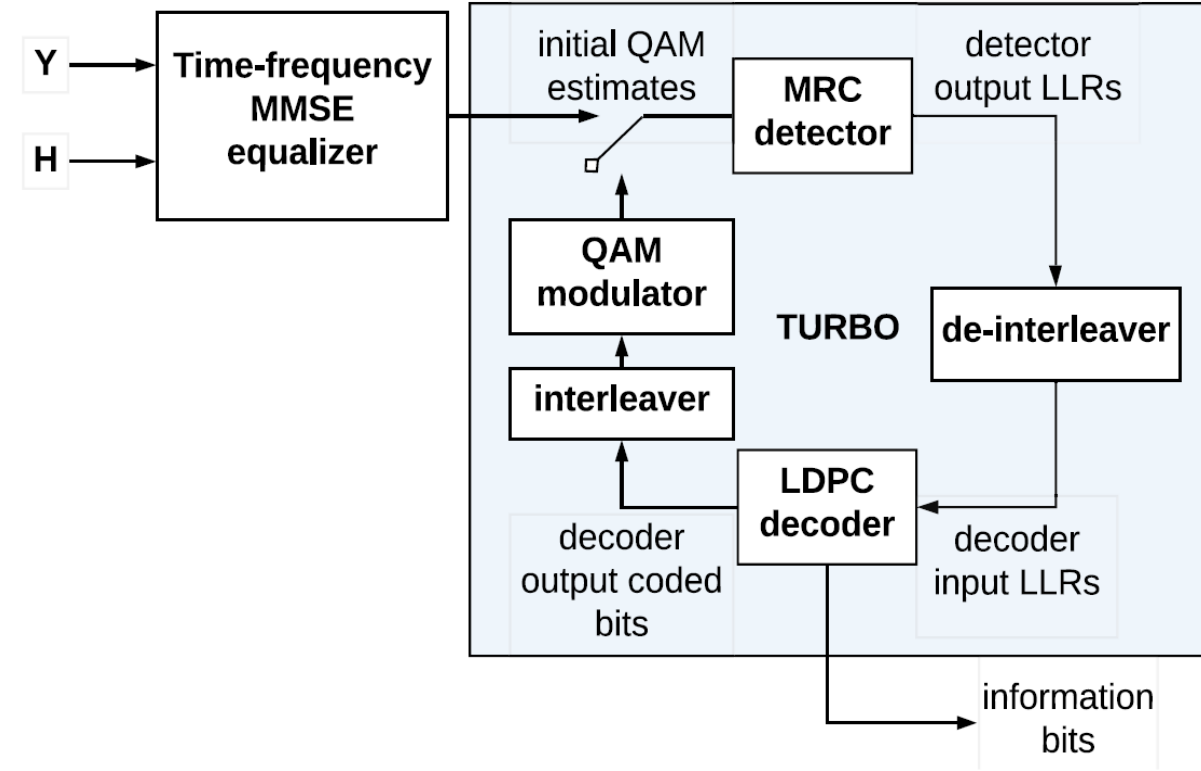


Fig. 8. OTFS iterative rake turbo decoder operation.

• Algorithmes de Détection

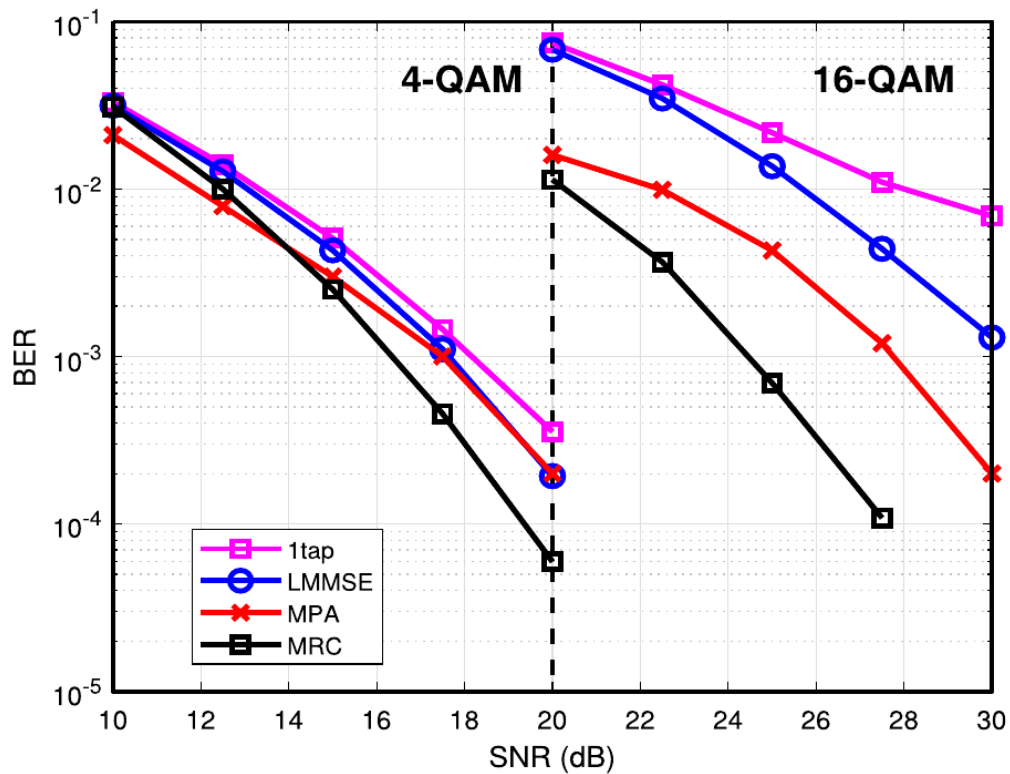


FIGURE 6.4 The BER performance of OTFS using 4-QAM and 16-QAM signaling and different detectors for a frame size of $N = M = 64$ (MP has 10 iterations for 4-QAM and 15 iterations for 16-QAM; MRC has 5 iterations for 4-QAM and 10 for 16-QAM).

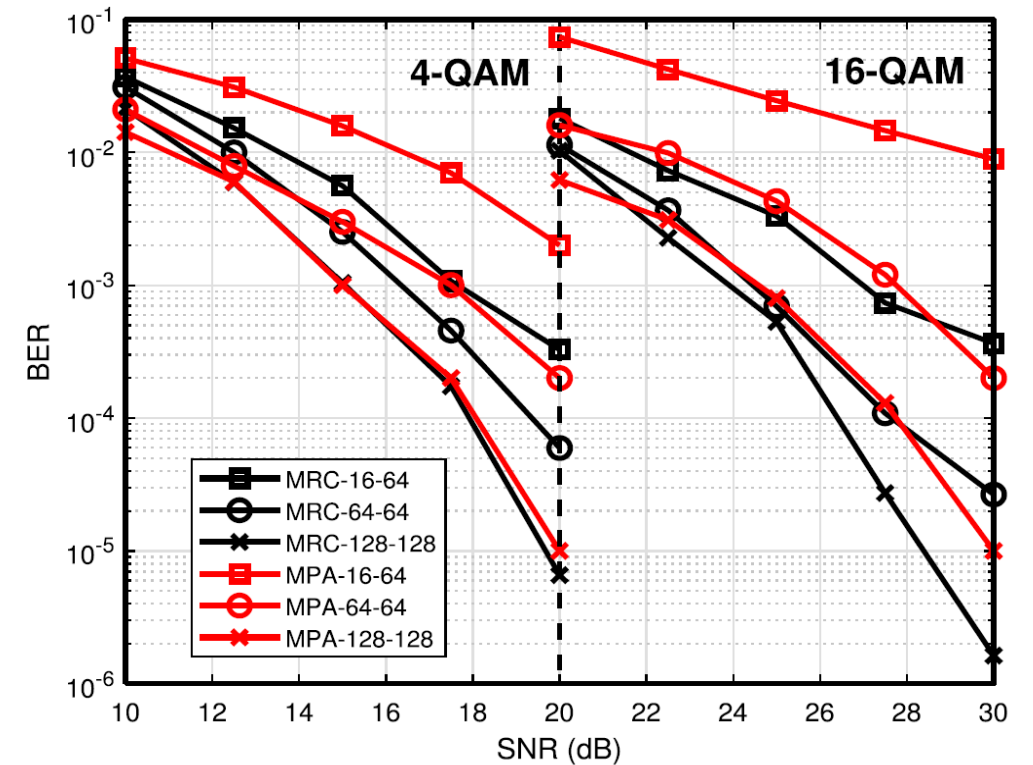


FIGURE 6.5 The BER performance of OTFS using MP and MRC detectors in the EVA channel model with different frame sizes (MP has 10 iterations for 4-QAM and 15 iterations for 16-QAM; MRC has 5 iterations for 4-QAM and 10 for 16-QAM).

Sur canal EVA (Extended Vehicular) avec spectre Doppler de Jakes, $v_{max} = 120 \text{ km/h} @ f_c = 4\text{GHz} \Leftrightarrow v_{max} = 444\text{Hz}$ (BER non codé).

Algorithmes de Détection

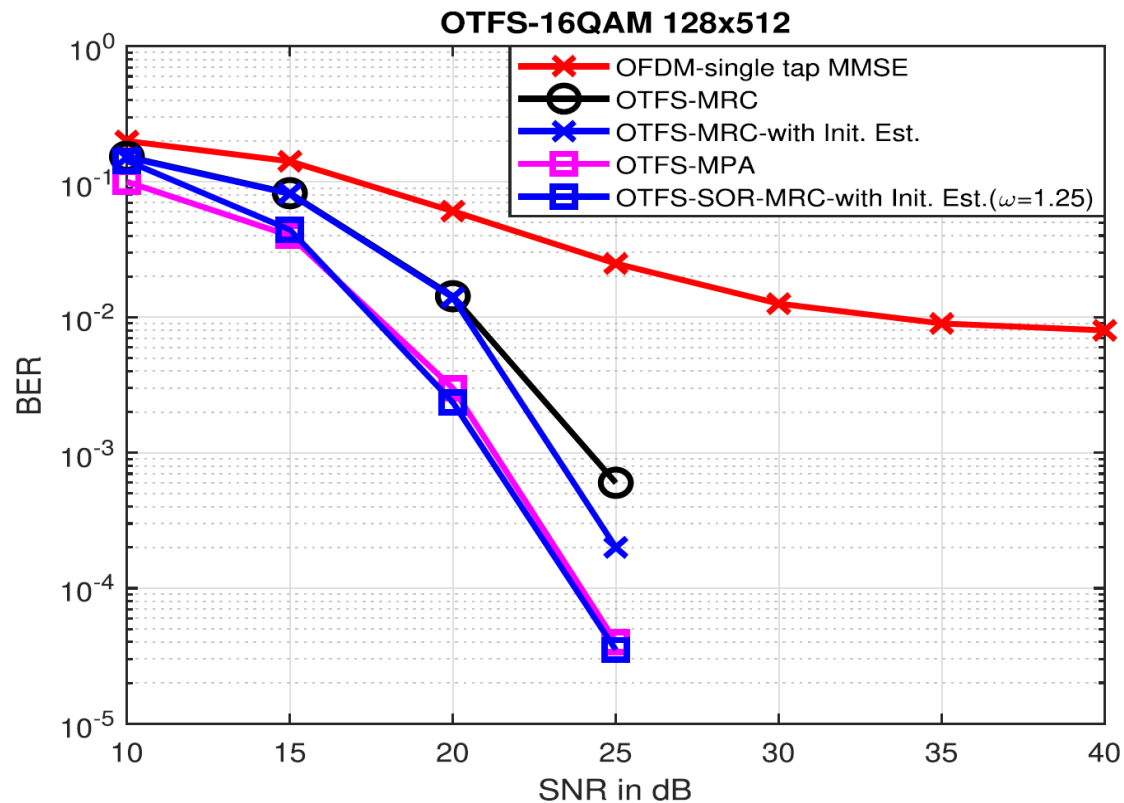


Fig. 10. Uncoded 16-QAM BER Plot : MRC vs MPA vs MMSE-OFDM.

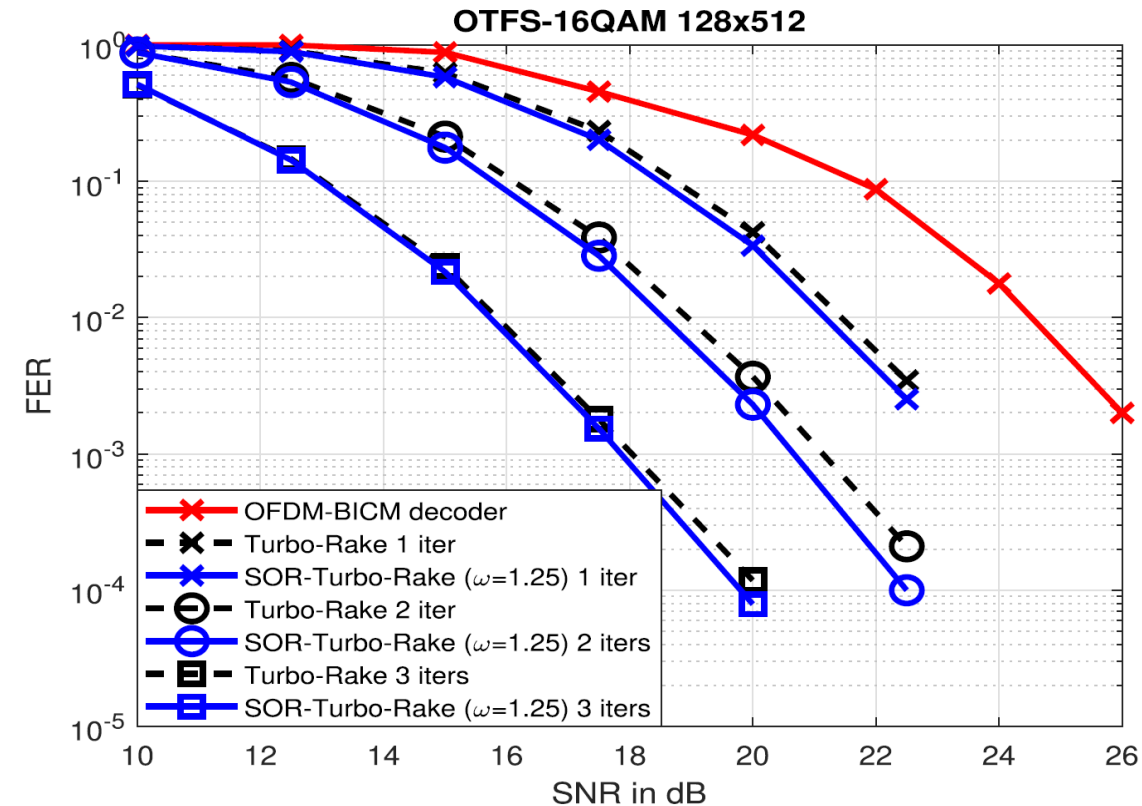


Fig. 11. Turbo 16-QAM FER Plot: MRC vs BIC-MMSE-OFDM.

Tiré de Thaj & Viterbo, *IEEE, Trans Veh. Tech* 2020

⁷The EVA channel power-delay profile (with a maximum speed = 120 km/hr) is given by [0, -1.5, -1.4, -3.6, -0.6, -9.1, -7.0, -12.0, -16.9] dB with excess delay taps $\mathcal{L}' = \mathcal{L} = \{0, 1, 2, 3, 4, 5, 8, 13, 19\}$ normalized to the delay resolution $1/(M\Delta f)$ of an OTFS grid with bandwidth $M\Delta f$, where $M = 512$ and $\Delta f = 15$ kHz.

- Turbo-égalisation : calcul des paramètres de l'égaliseur (entrées soft demapper)

$$\hat{x}_{m,n} = \mathbf{p}_{m,n}^H (\mathbf{y}_m - \mathbf{H}_m \bar{\mathbf{x}} + \mathbf{h}_{m,n} x_{m,n})$$

$$\mathbf{p}_{m,n} = (\mathbf{A}_m + (1 - v_{m,n}^2) \mathbf{h}_{m,n} \mathbf{h}_{m,n}^H)^{-1} \mathbf{h}_{m,n}$$

$$(\mathbf{A}_m + (1 - v_{m,n}^2) \mathbf{h}_{m,n} \mathbf{h}_{m,n}^H)^{-1}$$

$$= \mathbf{A}_m^{-1} - \frac{1 - v_{m,n}^2}{1 + (1 - v_{m,n}^2) \lambda_{m,n}} \lambda_{m,n} \mathbf{A}_m^{-1} \mathbf{h}_{m,n} \mathbf{h}_{m,n}^H \mathbf{A}_m^{-1}$$

$$\Rightarrow \mathbf{p}_{m,n} = \frac{1}{1 + (1 - v_{m,n}^2) \lambda_{m,n}} \mathbf{A}_m^{-1} \mathbf{h}_{m,n}$$

Avec $v_{m,n}^2$ = variance a priori du symbole $x_{m,n}$

$$\lambda_{m,n} = \mathbf{h}_{m,n}^H \mathbf{A}_m^{-1} \mathbf{h}_{m,n}$$

$$= \mathbf{h}_{m,n}^H (\sigma^2 \mathbf{I}_N + \mathbf{H}_m \mathbf{V}_{xx} \mathbf{H}_m^H)^{-1} \mathbf{h}_{m,n}$$

En supposant

$$p(\hat{x}_n | x_n = \alpha_k) = \mathcal{CN}(\mu_{n,k}, \sigma_{n,k}^2)$$

On a

$$\mu_{n,k} = \frac{\lambda_{m,n}}{1 + (1 - v_{m,n}^2) \lambda_{m,n}} \alpha_k = \gamma_n \alpha_k$$

$$\sigma_{n,k}^2 = \sigma_n^2 = \frac{\lambda_{m,n} (1 - v_{m,n}^2 \lambda_{m,n})}{(1 + (1 - v_{m,n}^2) \lambda_{m,n})^2}$$

Dans le calcul du LLR

$$\beta_n = \frac{1 + (1 - v_{m,n}^2) \lambda_{m,n}}{1 - v_{m,n}^2 \lambda_{m,n}} = \frac{\gamma_n}{\sigma_n^2}$$