

Research on Non-Terrestrial Networks: Design Tools, Regulation, and Cooperation

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Structure of the Presentation

- Non Terrestrial Communication Networks
 - NTN Classes
 - NTN Functionalities
 - NTN Communication Engineering Challenges
- System Level Analysis Methodology
- Illustrations
- NTN Impacts
- Ongoing Research, Actions

NTN CLASSES

- **UAVS** and **HAPS** Fleets
 - **Unmanned Aerial Vehicles**: Operate at low to medium altitudes, below commercial air traffic
 - **High-Altitude Platform Stations**: Operate in the stratosphere, (17 to 50 km high), above commercial air traffic
- **LEO** and **MEO** Satellite Constellations
 - **Low Earth Orbit**: Operate at altitudes from 350 to 2000 km, very fast speed (7 km/s), very low latency
 - **Medium Earth Orbit**: Operate at altitudes from 2000 to 30 000 km, lower speed, higher latency

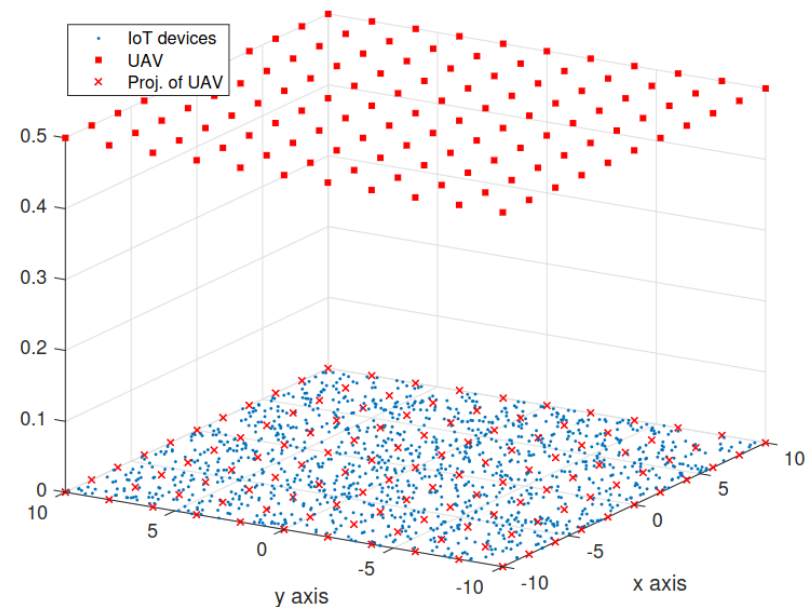
NTN: THREE BASIC SCENARIOS

- Poisson - Binomial
- Cox on Lines/Orbits
- Grid/Walker

UAVs AND HAPS ON GRIDS

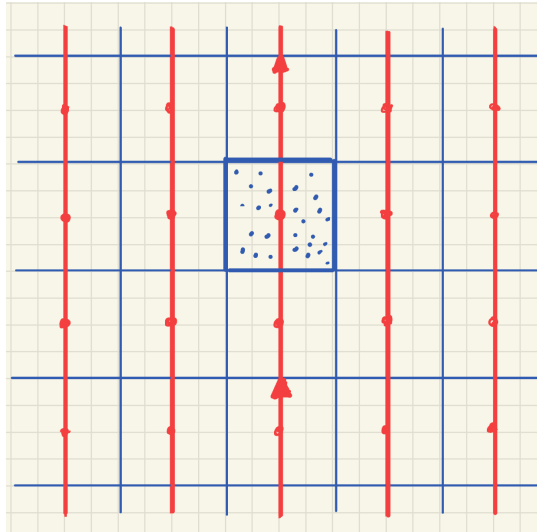
$\mathbb{Z}^2$ case: [Choi, FB & de Veciana, IEEE TWC, 20]

- Square grid of UAVs or HAPS with
 - * constant and equal speeds
 - * constant and equal altitudes
- IoT devices forming a homogeneous planar Poisson Point Process



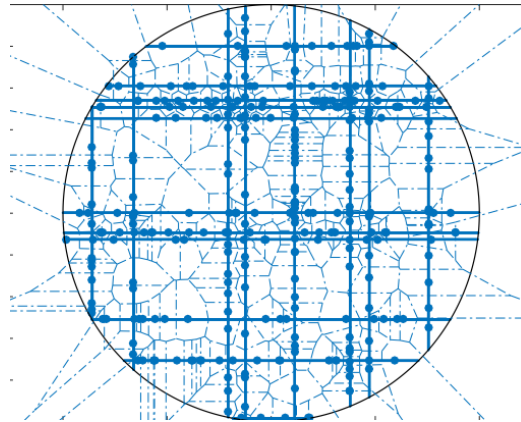
UAVs AND HAPS ON GRIDS (*continued*)

- If altitude is the same for all UAVs/HAPS
- For all devices, the nearest UAV/HAPS is also the nearest of the fleet projected on the user plane

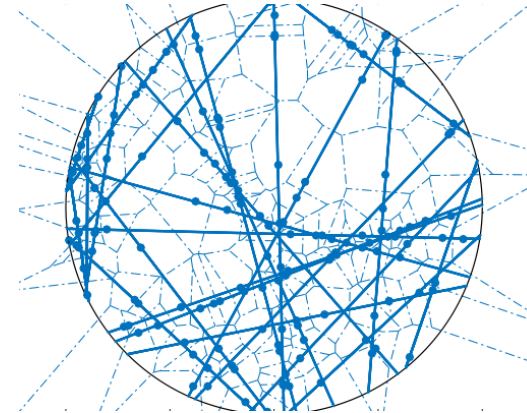


UAVs AND HAPS ON RANDOM TRAJECTORIES

In the UAV Plane, in place of the $\mathbb{Z}^2$ -grid: Cox on Lines



Linear Poisson HAPS/UAVS
on random GRID lines

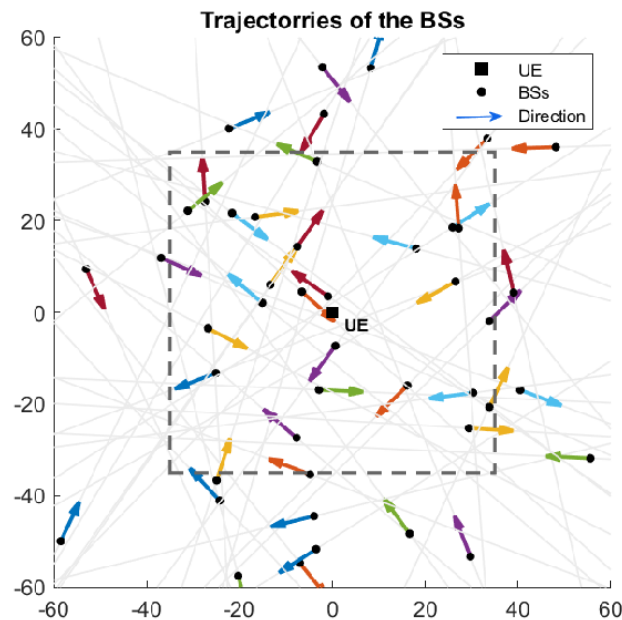


Linear Poisson HAPS/UAVS
on isotropic random lines

[Choi & FB, IEEE Trans. Veh. Tech. 18]

UAVs AND HAPS WITH INDEPENDENT RANDOM MOTIONS

Poisson Example with Independent Mobility
[Jhawar, FB, ArXiv 25, 26]



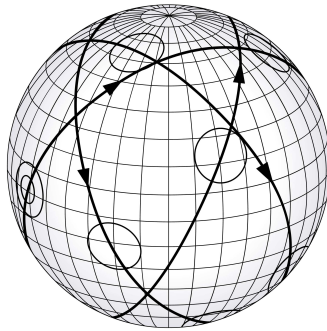
- In the HAPS/UAV Plane
 - Poisson initial locations
 - Constant speed
 - I.i.d. uniform directions
- In the user plane
 - Poisson initial locations
 - No motion

LEO/MEO SATELLITE CONSTELLATIONS

- Constellations
 - **LEO**: Low Earth Orbit
 - **MEO**: Medium Earth Orbit
- Grid type: **Walker's classification** – simplest version
 - Circular orbits
 - All altitudes equal
 - Periodic arrangement of orbits
 - Periodic arrangement of satellites
- **Lots of variants**: multi-altitudes, mixture LEO, MEO

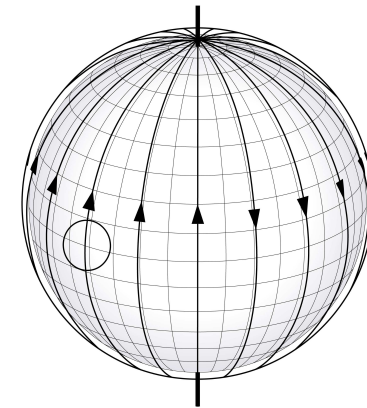
LEO/MEO SATELLITE CONSTELLATIONS (*continued*)

Periodic arrangement of orbits



Walker Delta basis of Starlink and Galileo

- Orbital planes: all same inclination
- Ascending points: period. on 360° of Equator

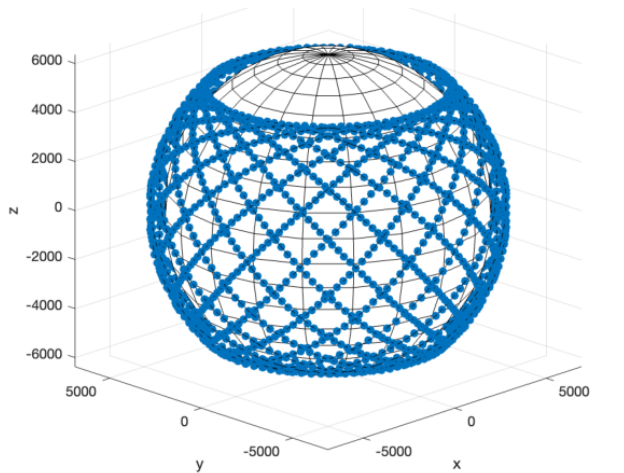


Walker Star basis of Oneweb

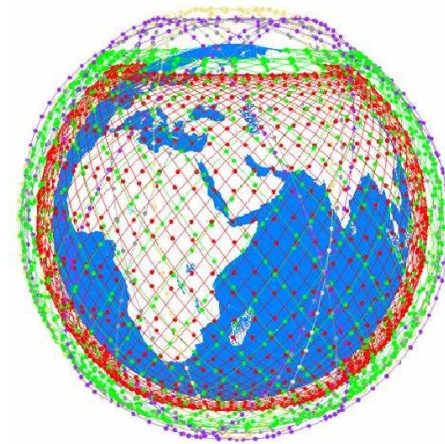
- Inclinations of orbits: $\sim 90^\circ$ (quasi polar)
- Orbital planes: periodic on 180° of equator

LEO/MEO SATELLITE CONSTELLATIONS (*continued*)

+ Periodic arrangement of satellites on each given orbit



Walker Δ – Single Altitude

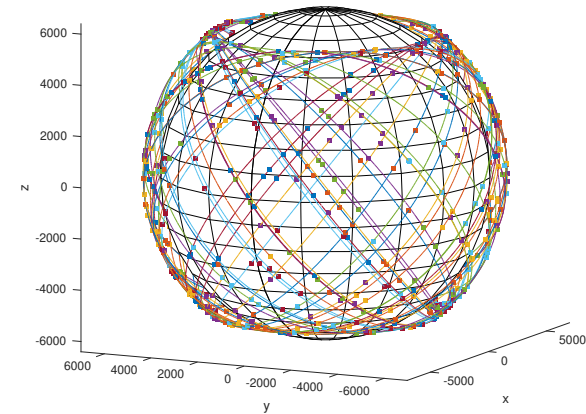


Walker Δ – Multi Altitude

FURTHER CONSTELLATION SCENARIOS

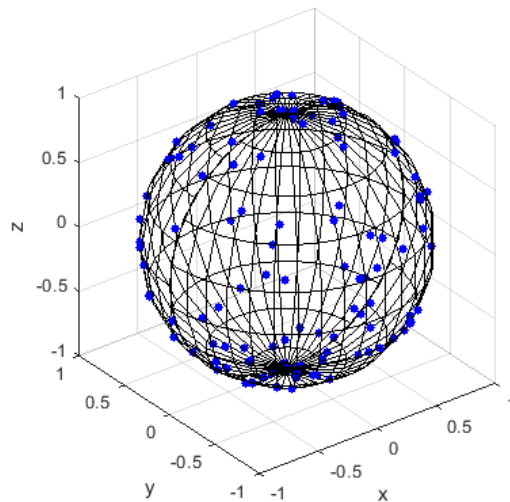
Cox on Orbits

- Poisson orbits with fixed inclinations and random ascending points
- Poisson satellites on orbits
- Can be seen as a partial deployment of Walker



[Choi & FB, IEEE Tr. Com. 25]

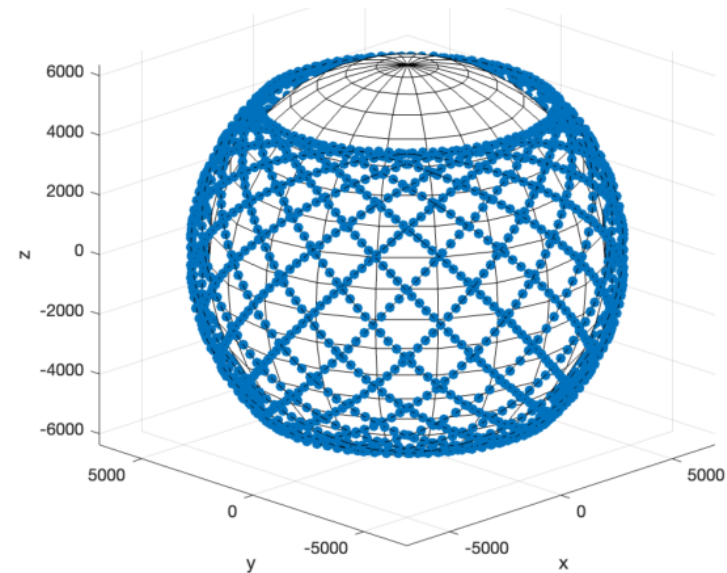
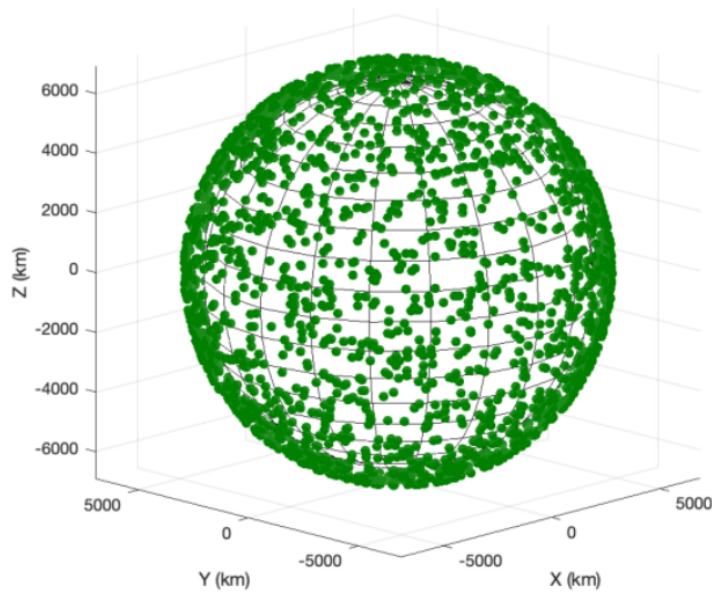
FURTHER CONSTELLATION SCENARIOS



[Okati et al.
IEEE Trans. Comm. 20]

- Binomial p.p. with satellites uniformly distributed on a sphere
- Each satellite has its own independent orbit
- The modulus of speed is the same for all satellites

NTN SCENARIOS COMPARED



FUNCTIONALITIES of CONSTELLATIONS

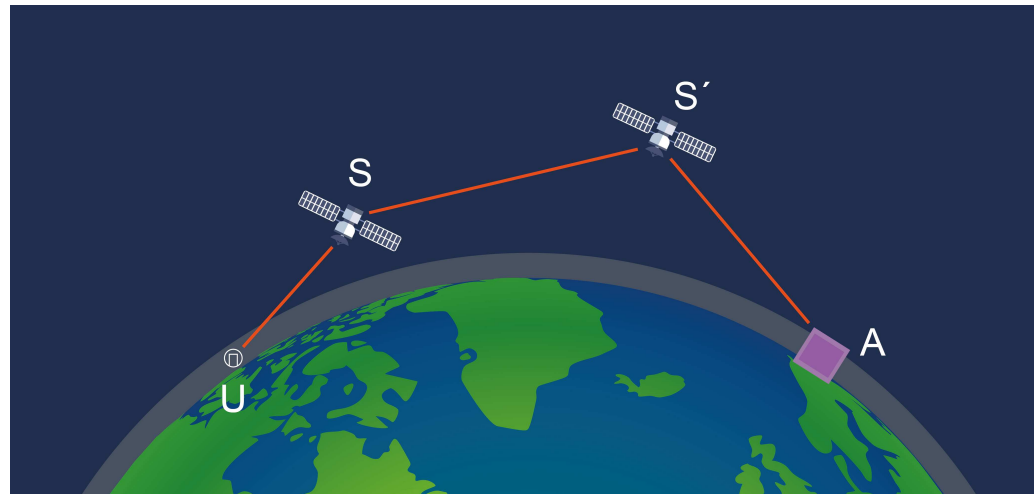
- **Communications: Broadband**
few constellations, each with a large number of satellites
- **Communications: IoT**
many constellations, with a smaller number of satellites
- **Earth observation**
many constellations, each with fewer satellites
- **Geolocalisation**
few constellations, each with few satellites

Future: Data center constellations....

SATELLITE CONSTELLATIONS FOR BROADBAND COMMUNICATIONS

- Coverage solution which is
 - **universal** (latitude constraints for some configurations)
 - **without interruptions** (if many satellites)
 - **broadband** (constraints on number of users per satellite)
 - **low latency** (because low orbits)
 - **resilient** (distributed system)

COMMUNICATIONS BETWEEN SATELLITES



Credits hal-04626677v1

U: *UE*, **S:** *satellite*, **A:** *gateway station*

Examples of S-U and S-A links:

Ku (10,7-14,5 GHz), Ka (17,3-30 GHz), V (37-50,4 GHz) & 5G Bands

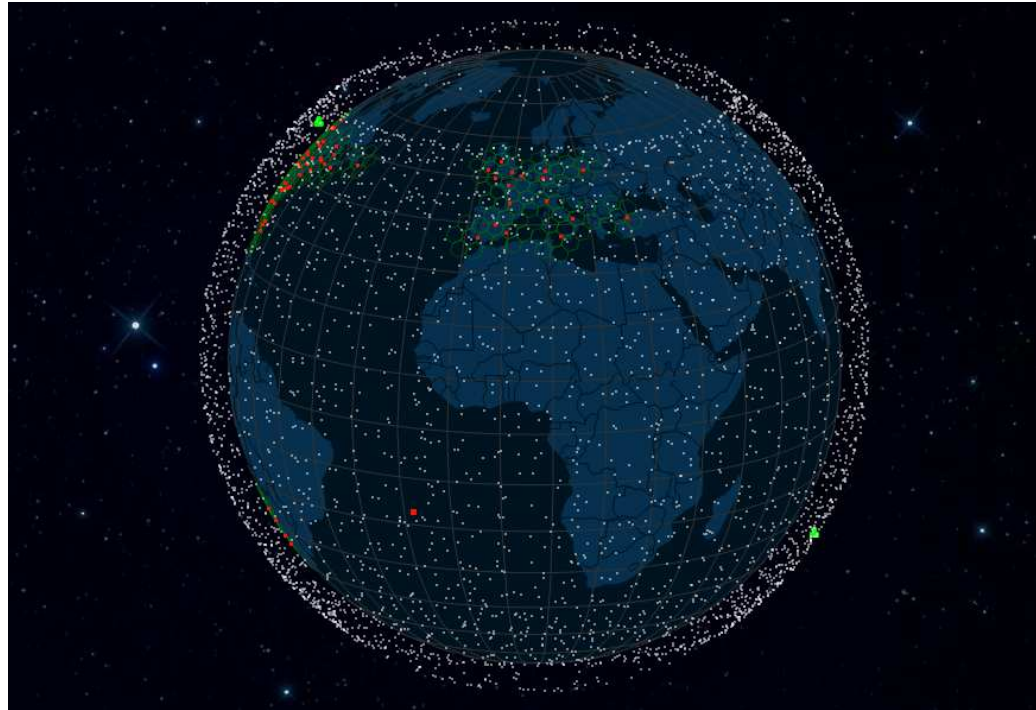
Inter satellite links: *Laser*

OLD, NEW & FUTURE CONSTELLATION

Broadband Communications - Examples

- **Iridium** (USA) ★
N < 100
- **Globalstar** (USA) ★
N=48
- **Oneweb** (UK,F) ★
N=624
- **O3B** (USA), equator.,
N ~ 30
- **Starlink** (USA) △
N=10→42K
- **Kuiper** (USA) mix
N=10K
- **IRIS²** (UE) mix
N=300
- **Guowang** (China)
N=13K
- **Thousand Sails** (China)
- **Hongyan** (China)

THE STARLINK CONSTELLATION



~ 10 K satellites end of 2025, authorization for 42 K
Altitude : between 350 and 550 km

SOVEREIGNTY ISSUES: OBSERVATION

- The race is on for **space and frequency occupation**
- The only current regulatory mechanism for the launch of broadband constellations is that of **the ITU**
- Communications networks based on these constellations are effectively, if not legally, exempt from all the rules imposed by States on terrestrial operators operating on their soil.
 - * mechanisms for **frequency allocation**
 - * **confidentiality rules** on conversations or data
 - * rules for **location of network cores**
- **As of today, these networks can do without anchor stations in the countries they cover**

NTN COMMUNICATION ENGINEERING CHALLENGES

■ Spectral Efficiency

- Coverage
- Spectral efficiency
- TN–NTN interference
- Antennas, RIS enhanced

■ Mobile Base Stations

- Doppler Shift
- Beam management
- Handover
- Multi-hop routing

■ TN-NTN Interplay

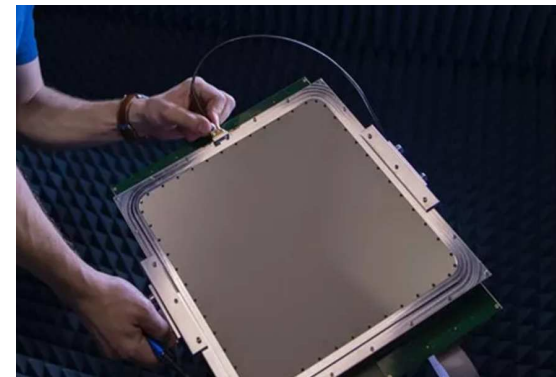
- 5G offloading
- Vertical handovers

■ NT Edge Computing

- Regenerative Satellites
- Combination of
 - * observation,
 - * computing and
 - * communication
- **Space Based Internet**

WHY ARE THESE CHALLENGES? : PHYSICAL LAYER

- Far Away BSs
 - Low power signal
 - Need for new UE antennas
 - Need for new waveforms



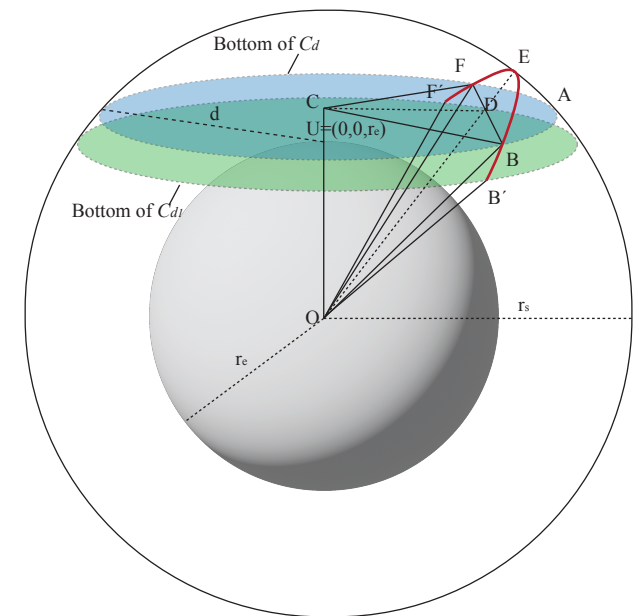
Greenerwave's RIS based
UE Antenna

Further RIS based solutions
to be discussed later

WHY ARE THESE CHALLENGES? : SPECTRAL EFFICIENCY

■ Spherical Geometry

- New **coverage** properties
- New **visibility cap** constraints
- New **def. of association cells**
- New **structure of interference**
- New **spectral efficiency**



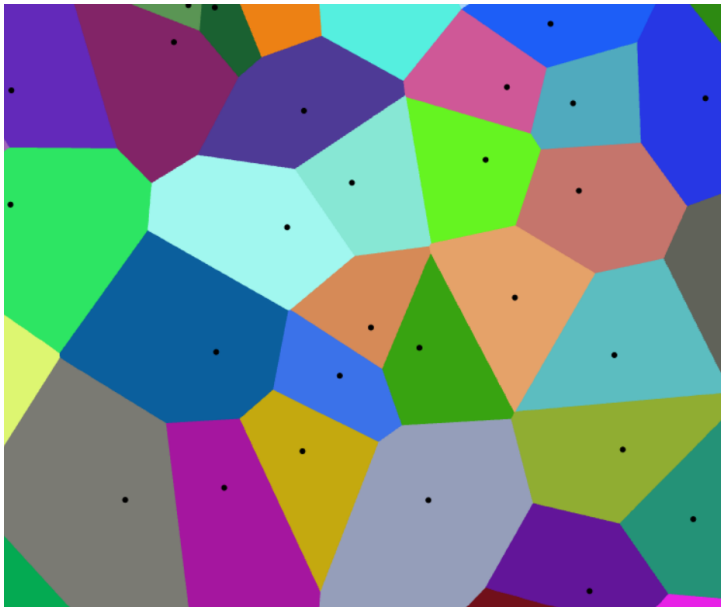
Visibility Cap

Visibility Arc of an Orbit

WHY ARE THESE CHALLENGES? : BS MOBILITY

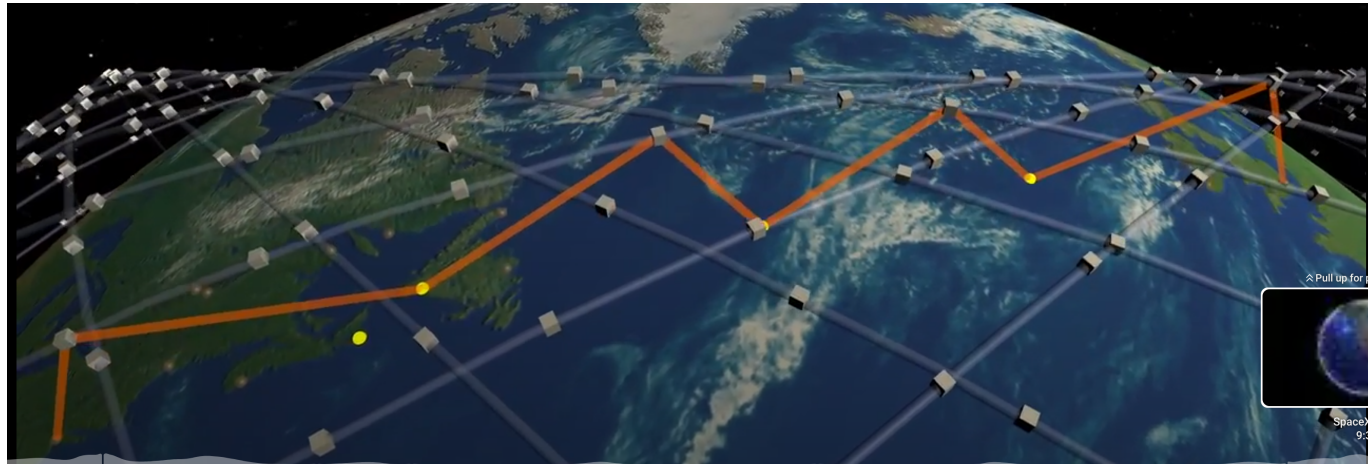
- Dynamical Moving BSs
 - Lower coherence times (Doppler)
 - Higher handovers frequency
 - New association rules
 - New multihop routing mechanisms
 - New types of handovers

DYNAMICS OF GEOMETRY – HANDOVERS IN THE UAV/HAPS CASE



- Dynamics of single altitude stations and their service areas
- Straight motion with random direction and constant speed case
- Projection in the Earth plane
- Credit: [A. Beutel, WebGL]

DYNAMICS OF GEOMETRY – ROUTING IN THE WALKER CONSTELLATION CASE



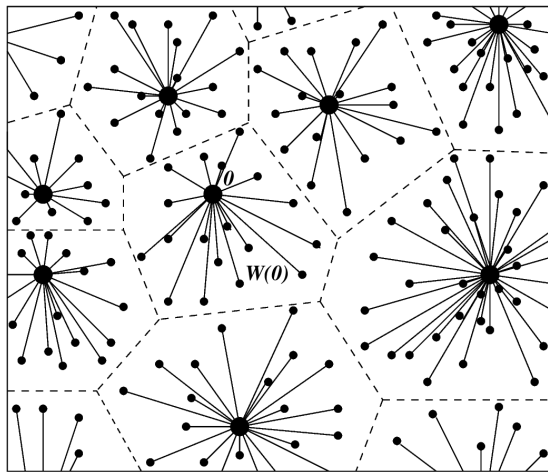
Dynamics of Shortest Path Routing
Walker Constellation plus Terrestrial Gateways

Credit: [M. Handley (UCL) Youtube]

Structure of the Presentation

- Non Terrestrial Communication Networks
- System Level Analysis Methodology
 - Stochastic Geometry
 - Dynamical Systems
- Illustrations
- NTN Impacts
- Ongoing Research, Actions

STOCHASTIC GEOMETRY FOR PLANAR CELLULAR NETWORKS – RECAP



Terrestrial Base Stations,
UEs, cells and wireless
links

- **Base stations** (BSs) arranged according to some homogeneous Poisson point process in $\mathbb{R}^2$
- **User Equipments** (UEs) located according to some independent stationary point process

Each UE served by closest BS → **Poisson Voronoi Cells**

SHANNON RATE IN POISSON-VORONOI CELLULAR NETWORKS WITH RAYLEIGH FADES

- SINR experienced by typical UE: $\text{SINR} := \frac{S}{I+N}$
 - **S: Signal power**: stems from closest BS
 - **I: Interference power**: from BSs outside Voronoi cell of typical UE
 - **N**: thermal noise power
- Shannon rate offered to typical UE: $\mathcal{T} \sim B \log(1 + \text{SINR})$
- **Question**: Law of the Shannon rate offered to typical user

COVERAGE/SHANNON RATE

$$\mathbf{p}_c(\mathbf{T}, \lambda, \alpha) = \mathbb{P}_{\mathbf{u}}^0[\mathbf{SINR} > \mathbf{T}] = \mathbb{P}_{\mathbf{u}}^0[\text{Shannon rate} > \mathbf{B} \log(\mathbf{1} + \mathbf{T})]$$

- As in statistical physics, this coincides with **empirical ensemble averages**
 - Average fraction of users who achieve SINR at least $\mathbf{T}$
 - Average fraction of the network area in “ $\mathbf{T}$ -coverage”

PLANAR POISSON VORONOI SHANNON THEOREM

■ Theorem [J. Andrews, FB & R. Ganti, IEEE Tr.Comm. 11]

- Power law path loss : $l(\mathbf{r}) = \mathbf{r}^\alpha$, $\alpha > 2$, path loss exp.
- Rayleigh fading model with mean $\frac{1}{\mu}$
- Service by the closest BS (best SNR)

$$p_c(\mathbf{T}, \lambda, \alpha) = \pi \lambda \int_0^\infty e^{-\pi \lambda \mathbf{v}(1+\rho) - \mu \mathbf{T} \mathbf{N} \mathbf{v}^{\alpha/2}} d\mathbf{v} \quad \text{with} \quad \rho = \mathbf{T}^{\frac{2}{\alpha}} \int_{\mathbf{T}^{-\frac{2}{\alpha}}}^\infty \frac{1}{1 + \mathbf{u}^{\alpha/2}} d\mathbf{u}$$

- Step 1 : Poisson–Voronoi : distance to closest BS: Rayleigh distr. $\rightarrow \mathbf{S}$
- Step 2 : Poisson–Voronoi : Poisson shot-noise outside a ball $\rightarrow \mathbf{I}$
- Step 3 : Poisson–Voronoi–Shannon : law of SINR and Shannon rate

■ $\rightarrow$ system level analysis of spectral efficiency

PROOF

■ Step 1: Poisson-Voronoi

$$\begin{aligned}
 p_c(\mathbf{T}, \lambda, \alpha) &= \mathbb{P}[\mathbf{SINR} > \mathbf{T}] \\
 &= \int_{r>0} \mathbb{P}[\mathbf{SINR} > \mathbf{T}] f_r(r) dr \\
 &= \int_{r>0} \mathbb{P} \left[\frac{\mathbf{S} r^{-\alpha}}{\mathbf{N} + \mathbf{I}_r} > \mathbf{T} \right] e^{-\pi \lambda r^2} 2\pi \lambda r dr \\
 &= \int_{r>0} e^{-\pi \lambda r^2} \mathbb{P}[\mathbf{S} > \mathbf{T} r^\alpha (\mathbf{N} + \mathbf{I}_r)] 2\pi \lambda r dr
 \end{aligned}$$

$\mathbf{I}_r$: interference power given the closest BS is at distance r

PROOF (continued)

■ Step 2: **Rayleigh**. Since $S \sim \exp(\mu)$,

$$\mathbb{P}(S > \mathbf{Tr}^\alpha(\mathbf{N} + \mathbf{I}_r)) = \mathbb{E}[\exp(-\mu \mathbf{Tr}^\alpha(\mathbf{N} + \mathbf{I}_r))] = e^{-\mu \mathbf{Tr}^\alpha \mathbf{N}} \mathcal{L}_{\mathbf{I}_r}(\mu \mathbf{Tr}^\alpha)$$

with $\mathcal{L}_{\mathbf{I}_r}(\mathbf{s})$ the **Laplace transform of the interference**
 Thus

$$\mathbf{p}_c(\mathbf{T}, \lambda, \alpha) = \int_{r>0} e^{-\pi \lambda r^2} e^{-\mu \mathbf{Tr}^\alpha \mathbf{N}} \mathcal{L}_{\mathbf{I}_r}(\mu \mathbf{Tr}^\alpha) 2\pi \lambda r dr$$

PROOF (*continued*)

■ Step 3: Interference as shot noise field

$$\mathcal{L}_{\mathbf{I}_r}(\mathbf{s}) = \exp \left(-2\pi\lambda \int_{\mathbf{r}}^{\infty} (1 - \mathcal{L}_{\mathbf{F}}(\mathbf{s}\mathbf{v}^{-\alpha})) \mathbf{v} d\mathbf{v} \right)$$

with $\mathcal{L}_F$ the Laplace Transform of the general fading

■ plus immediate computations

EXAMPLE OF USE: SINR WALL

- **System Level Question:** When does BS densification decrease spectral efficiency?
- When $N = 0$, for power law attenuation, constant spectral efficiency for all densities !

$$p_c(\mathbf{T}, \lambda, \alpha) = \frac{1}{1 + \rho(\mathbf{T}, \alpha)}$$

When in addition, $\beta = 4$: $p_c(\mathbf{T}, \lambda, 4) = \frac{1}{1 + \sqrt{\mathbf{T}}(\pi/2 - \arctan(1/\sqrt{\mathbf{T}}))}$

- When $N = 0$, for all bounded decreasing attenuation functions

$$p_c(\mathbf{T}, \lambda, \alpha) \rightarrow 0 \text{ as } \lambda \rightarrow \infty$$

[A. Alammouri, FB & J. Andrews, IEEE Tr. I.T., 19]

MANY VARIANTS STUDIED SINCE 2011

- Beyond Poisson: Determinantal, Grids
- **More general fading**
- More general attenuation function
- Power control
- Treating interference as signal
- Spatial reuse
- **Beamforming**

BEAMFORMING

- Received signal power of the form

$$pG_t G_r H d^{-\alpha}$$

- G_t : antenna gain of transmitter
- G_r : antenna gain of receiver
- H : Nakagami
- α : path loss exponent
- d : distance

- Different antenna gains for

- desired transmissions: G_t, G_r
- interfering transmissions: g_t, g_r

Structure of the Presentation

- Non Terrestrial Communication Networks
- System Level Analysis Methodology
- Illustrations
 1. Spectral Efficiency in NTN
 2. Routing in Walker Constellations
 3. Handovers in UAV and HAPS Networks
 4. Doppler-Shannon Association in UAV and HAPS NTN
- NTN Impacts
- Ongoing Research, Actions

1. SPECTRAL EFFICIENCY IN NTN: MAIN SG RESULTS

- Spatial averages for Poisson, Cox and Grid/Walker scenarios:
 - Closest BS distance
 - No visible BS probability
 - Mean number of visible BS
 - Coverage probability
 - Distribution of Shannon rate
- Illustrations:
 - UAV/HAPS: Poisson and Cox
 - Constellations : Cox and Walker

POISSON-VORONOI-SHANNON FOR UAVs/HAPS

- Same principles and same types of results obtained by changing the response function from $l(\mathbf{r}) = r^\alpha$ to

$$l_h(\mathbf{r}) = (r^2 + h^2)^{\frac{\alpha}{2}}$$

with h the altitude of UAVs/HAPS

- Probability of coverage given the projection of the closest BS is at distance r :

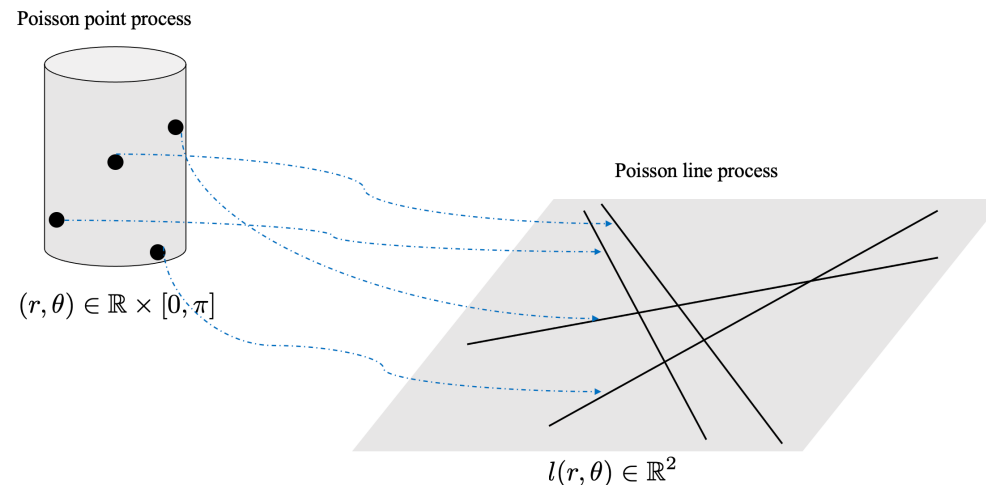
$$\begin{aligned} \mathbb{P}(\mathbf{S} > \mathbf{T}r^2 + \mathbf{h}^2)^{-\alpha/2}(\mathbf{N} + \mathbf{I}_r)) &= \mathbb{E}[\exp(-\mu \mathbf{T}(r^2 + h^2)^{-\alpha/2}(\mathbf{N} + \mathbf{I}_r))] \\ &= e^{-\mu \mathbf{T}(r^2 + h^2)^{-\alpha/2} \mathbf{N}} \mathcal{L}_{\mathbf{I}_r}(\mu \mathbf{T}(r^2 + h^2)^{-\alpha/2}) \end{aligned}$$

with

$$\mathcal{L}_{\mathbf{I}_r}(\mathbf{s}) = \exp \left(-2\pi\lambda \int_r^\infty \frac{\mathbf{s}}{\mathbf{s} + \mu(\mathbf{r}^2 + \mathbf{h}^2)^{\alpha/2}} \mathbf{v} d\mathbf{v} \right)$$

COX-VORONOI-SHANNON FOR UAVs/HAPS

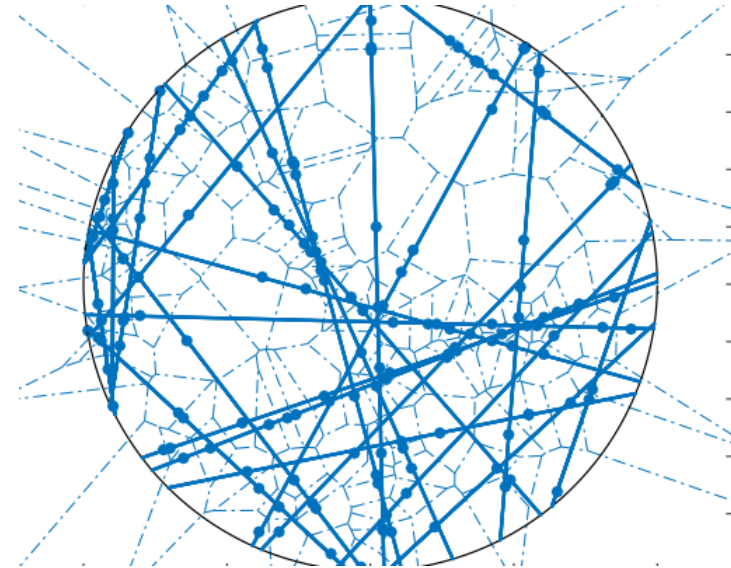
- Cox: Poisson random points with and intensity that is random (e.g. supported on random sets (lines))
- Trajectories as Poisson Lines, Satellites as Cox Point Process



1. A Poisson point process on a cylinder
2. Each point $\Rightarrow$ a line on the plane
3. Point process on lines $\Rightarrow$ Cox point process

COX-VORONOI FOR UAV/HAPS

- Trajectories:
Isotropic Poisson
line PP
- Linear BSs: linear
PPP on each line
- users: PPP in the
user plane



Note the qualitatively different shape and structure of the
(association) Voronoi cells

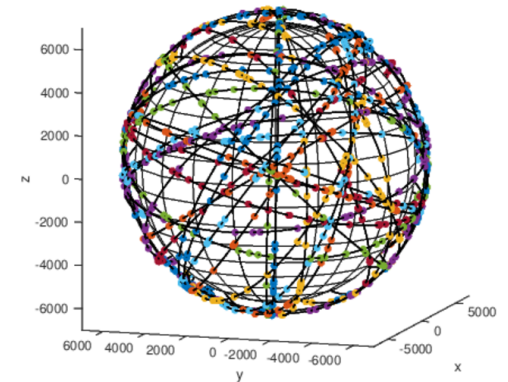
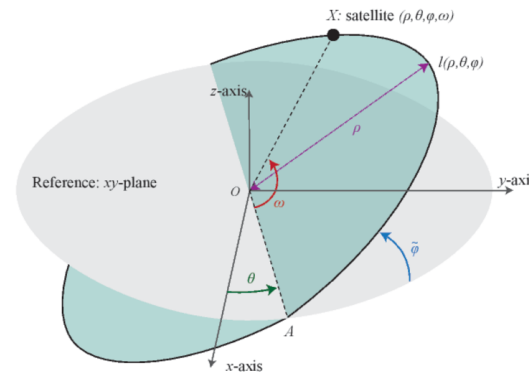
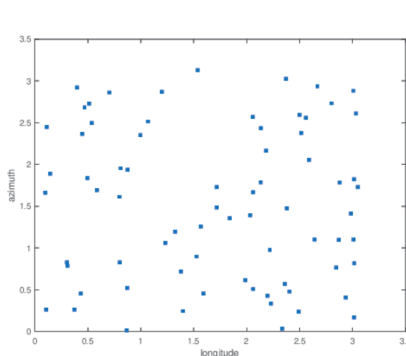
COX-VORONOI FOR UAV/HAPS (*continued*)

- The signal S stems from the closest point of the Cox PP
- The interference I comes from all other UAVs including those on the line that contains the closest (Palm bias)
- $\rightarrow$ integral from expressions for joint distribution of S and I , and hence **probability of coverage, spectral efficiency**
[C.-S. Choi, FB, IEEE Tr. Vehic. Netw. 2019]

ISOTROPIC SATELLITE COX PP

Orbital planes where LEO satellites are located

- (left) Poisson p.p. $\frac{\lambda \sin(\phi)}{2\pi}$ on a rectangle $\mathcal{R} = [\theta, \phi]$
- (mid) Map $\delta_{\theta, \phi} \Rightarrow$ an orbit $l(\theta, \phi)$, longitude θ , inclination ϕ
- (right) All Poisson points $\rightarrow$ all orbital planes



All performance metrics to come are averages w.r.t. the PPP of the left rectangle $\rightarrow$ isotropic orbits and constellations

KEY ISOTROPIC COX PERFORMANCE METRICS

- No discovery probability: Probability that users cannot find any visible satellite
- The coverage probability of the typical user (at $(0, 0, e)$)

$$\mathbb{P}(\text{SINR} > \tau) = \mathbb{P} \left[\frac{p G_t G_r H \|X_\star - (0, 0, e)\|^{-\alpha}}{N + \sum_{X_j \in \bar{\Psi} \setminus X_\star} p g_{t;X_j} g_{r;X_j} H_j \|X_j - (0, 0, e)\|^{-\alpha}} > \tau \right]$$

- * $X_\star$ association satellite
- * $\bar{\Psi}$ visible satellites
- * $G_t G_r$ antenna gains for desired transmissions
- * $g_{t,x} g_{r,x} = 1$ gains for interfering transmission at x

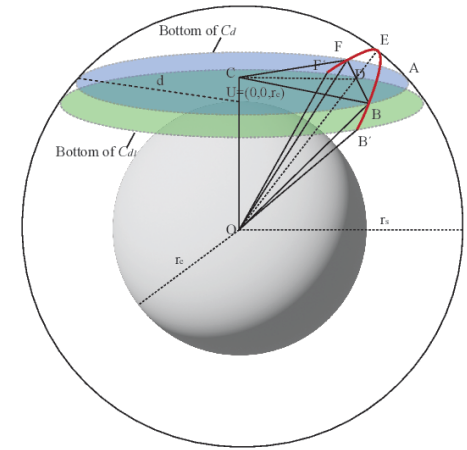
Here expectation is w.r.t. the Poisson orbits and Poisson PPs

SPHERICAL CAP AND ARC LENGTH

- Important for accounting visible and interfering satellites

- **Spherical cap:** subset of points of the sphere of radius r with distances less than d to the user

$$C_d := \{(\mathbf{x}, \mathbf{y}, \mathbf{z}) \in \mathbb{S} \mid \|(\mathbf{x}, \mathbf{y}, (\mathbf{z} - \mathbf{e}))\| \leq d\}$$



- The length of the intersection of C_d and the orbit $l(\theta, \phi)$ with inclination ϕ is

$$\widehat{BF} = 2r \arcsin \left(\sqrt{1 - \csc^2(\phi) \left(\frac{r^2 + e^2 - d^2}{2re} \right)^2} \right)$$

COX RESULTS: NUMBER OF VISIBLE SATELLITES

- **Proposition** [C.S. Choi, FB, IEEE Tr. Comm. 25]

The average number of visible satellite is

$$\bar{n} = \frac{\lambda\mu}{\pi} \int_0^{\bar{\varphi}} \cos(\varphi) \arcsin \left(\sqrt{1 - \cos^2(\bar{\varphi}) \sec^2(\varphi)} \right) d\varphi$$

Proof: for each orbit, **only a fraction of the orbit is visible**

$$\begin{aligned} \bar{n} &= \mathbb{E} \left[\sum_{l(\theta_i, \varphi_i) \in \mathcal{O}} \frac{\mu}{2\pi r_s} \text{length}(l(\theta_i, \varphi_i) \cap C_{d_1}) \right] \\ &= \frac{\lambda\mu}{\pi} \int_0^{\bar{\varphi}} \cos(\varphi) \arcsin \left(\sqrt{1 - (\mathbf{e}/\mathbf{r})^2 \sec^2(\varphi)} \right) d\varphi \end{aligned}$$

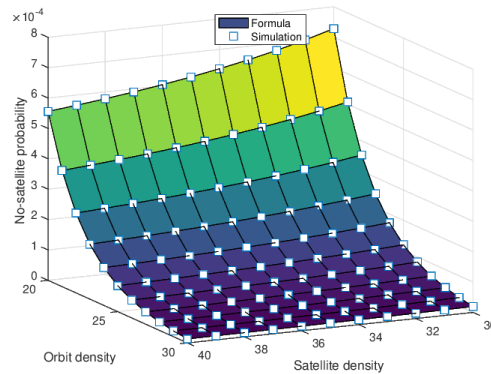
where $\bar{\varphi} = \arccos(\mathbf{e}/\mathbf{r})$

COX NO-SATELLITE PROBABILITY

■ **Theorem [C.S. Choi, F.B., Tr. Comm. 25]**

The no discovery probability of the typical user is given by

$$\mathbb{P}(\text{no discovery}) = e^{-\lambda \int_0^{\bar{\varphi}} \cos(\varphi) \left(1 - e^{-\mu \pi^{-1} \arcsin(\sqrt{1 - e^2 \sec^2(\varphi)/r^2})} \right) d\varphi}$$



– Constellation almost always visible for practical λ, μ

– For $\mu \gg 1$

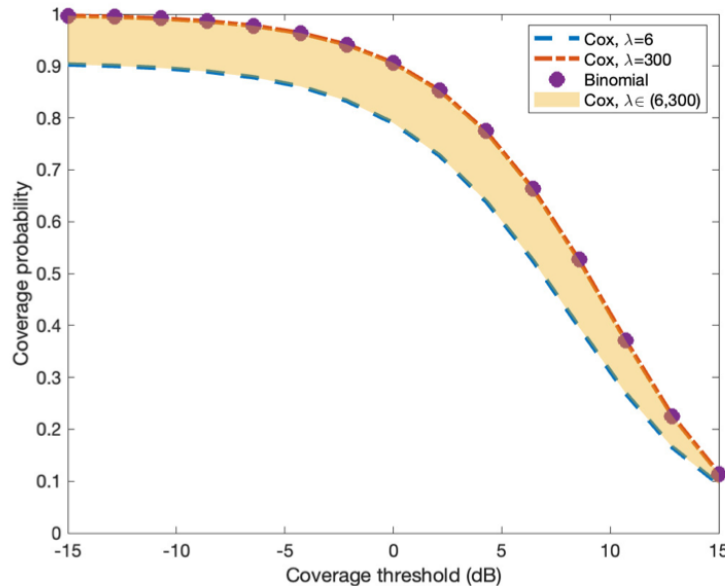
$$\mathbb{P}(\text{no discovery}) = \exp(-\lambda \sin(\bar{\varphi}))$$

no-satellite probability

COVERAGE PROBABILITY

- Extension of the planar Cox integral representation of probability of coverage based on closed forms for
 - the **distribution of the distance to the closest visible satellite** (if any)
 - the **Laplace transform of the interference** due to all satellites but the closest
- Extension of the planar Cox integral representation of the distribution of Shannon rate

COX COVERAGE PROBABILITY



- Probability of coverage as a function key variables $\lambda, \mu, r, \alpha, \dots$
- Wide range of results compared to Bernoulli/Poisson for fixed mean number of satellites

WALKER DELTA MODEL/NOTATION

- N_o number of orbits
- The inclinations of orbits are assumed to be all equal to ϕ
- The N_o ascending points are distributed regularly on $[0, 2\pi]$
- $l_i(\theta_i, \phi)$ the i -th orbit, with longitude angle θ_i
- If at $t = 0$, the longitude of the i -th orbit ascending point is

$$\theta_i = 2\pi i / N_o + \bar{\theta} \mod 2\pi$$

with $\bar{\theta} \sim \text{Uniform}(0, 2\pi / N_o)$ then the orbit process

$$\mathcal{O} = \bigcup_{i=1}^{N_o} l_i = \bigcup_{i=1}^{N_o} l(\theta_i, \phi)$$

has a distribution which is invariant w.r.t. Earth's spin

WALKER DELTA MODEL/NOTATION (*continued*)

- N_s number of satellites on each orbit
- Conditionally on these orbits, the phases of the N_s satellites on each orbit are periodically distributed over 2π
- The phase of the j -th satellite on the i -th orbit at $t = 0$ is

$$\omega_j = 2\pi j / N_s + \bar{\omega} \mod 2\pi, \quad j = 1, \dots, N_s$$

with $\bar{\omega} \sim \text{Uniform}(0, 2\pi / N_s)$ an independent random variable

- **Variant:** addition of offset $o_i = i \cdot \eta$ on orbit i
- Constellation P.P. at $t = 0$: $\Psi = \sum_{i=1}^{N_o} \psi_i = \sum_{i=1}^{N_o} \sum_{j=1}^{N_s} \delta_{\mathbf{X}_{i,j}}$
 - ψ_i is the satellite point process on orbit $l_i(\theta_i, \phi)$
 - $\mathbf{X}_{i,j}$ is the satellite of phase ω_j on $l_i(\theta_i, \phi)$

WALKER: STATISTICS OF DISTANCE TO NEAREST SATELLITE

- Let $D(l_u)$ be the distance from the typical user at latitude l_u to its nearest satellite. $D(l_u) = \infty$ if no satellite is visible from the typical user
- **Theorem** [C.S. Choi, F. B., Tr. Veh. Tech. 25]
For the Walker Delta, for $r - e < d < \sqrt{r^2 - e^2}$,

$$\mathbb{P}(D(l_u) > d) = \kappa \int_0^{\frac{2\pi}{N_o}} \int_0^{\frac{2\pi}{N_s}} \mathbf{1} \left\{ \frac{r^2 + e^2 - d^2}{2re} > \max \frac{\mathbf{X}_{i,j} \cdot \mathbf{u}}{\|\mathbf{X}_{i,j}\| \|\mathbf{u}\|} \right\} d\omega d\theta,$$

with $\kappa = N_o N_s / (4\pi^2)$

WALKER: STATISTICS OF DISTANCE TO NEAREST SATELLITE (continued)

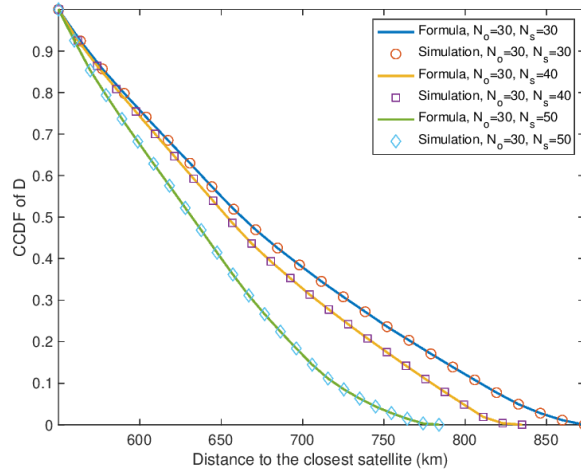


Fig. 2. The CCDF of random variable D_l . Here, $r = 6921$ km. The system-level simulation results confirm the accuracy of the derived formula.

- Thanks to the deterministic structure of the deployment of satellites, for latitudes l_u smaller than a threshold L , for N_o and N_s large enough, **there exists a constant $d_c(l_u) \leq \sqrt{r^2 - e^2}$ such that $P[D(l_u) \leq d] = 1$ for all $d > d_c(l_u)$**

For instance when $N_o = 30$ and $N_s = 50$, we have $\mathbb{P}(D(l_u) > d_c(l_u)) = 0$ for $d_c(l_u) \cong 775$ km, i.e., the association distance is for sure less than 775 km, whereas the max due to visibility is 2108 km

WALKER: LAPLACE TRANSFORM OF INTERFERENCE

■ **Theorem** [C.S. Choi, F. B., Tr. Veh. Tech. 25]

In the Walker delta case, the LT of interference for the typical user at u only depends on its latitude and is given by

$$\mathcal{L}_I(s) = \kappa \int_0^{\frac{2\pi}{N_o}} \int_0^{\frac{2\pi}{N_s}} e^{\left(\sum c_{i,j} \log \left(\mathcal{L}_H \left(\frac{\text{sp} G_{i,j}}{\|X_{i,j} - u\|^\alpha} \right) \right) \right)} d\bar{\omega} d\bar{\theta},$$

where

– $c_{i,j} = 1 \left\{ \frac{X_{i,j} \cdot u}{\|X_{i,j}\| \|u\|} \geq \frac{e}{r} \right\}$

– the $G_{i,j}$ stem from beamforming

■ → Distribution downlink Shannon rate at a given latitude

WALKER: LAPLACE TRANSFORM OF INTERFERENCE (continued)

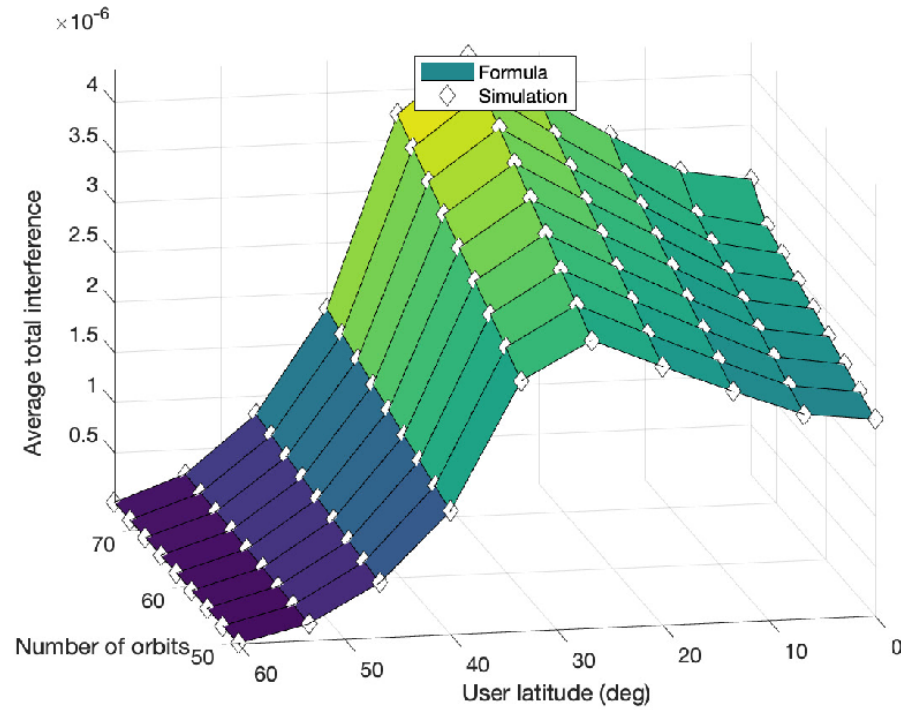


Fig. 3. The mean total interference. We use $\phi = 33^\circ$, $N_s = 75$, $B_w = 10$ MHz, $p = 20$ dBW, $g_t = 24$ dBi, $g_r = 1$, and $\eta = 5^\circ$ (or $d_g = 946$ km).

NTN DYNAMICS: MAIN QUESTIONS

- Dynamics of ephemerides seen by a typical UE
- For instance, dynamics of
 - Distance to closest satellite
 - Shortest route to a given gateway
 - Number of visible Satellites
 - SINR and Shannon rate

DYNAMICS: MAIN RESULTS

Mathematical machinery:

- Invariant measures
- Ergodicity, minimality
- Time Averages
- Statistics at given ephemeris events

Illustrated by

- Walker Constellation Ephemerides
- Poisson UAV/HAPS Ephemerides

DYNAMICAL SYSTEMS – RECAP

- $\mathcal{S}$ metric space with its Borel sigma algebra $\mathcal{B}$
- $(\mathbf{R}_t)_{t \geq 0}$ **measure preserving** dynamical system (flow) on $\mathcal{S}$:
 - Measurable map from $\mathcal{S}$ to $\mathcal{S}$
 - $\mathbf{R}_t \circ \mathbf{R}_s = \mathbf{R}_{t+s}$ for all s, t
 - Preserves μ , probability measure on $\mathcal{B}$: $\mu \mathbf{R}_t^{-1} = \mu$ for all t
- $((\mathbf{R}_t), \mu)$ is ergodic if the sigma algebra of $(\mathbf{R}_t)$ -invariant events contains only sets of measure 0 or 1
- **Birkhoff's theorem**: in the ergodic case, $\forall f : \mathcal{S} \rightarrow \mathbb{R}^+$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T f(\mathbf{R}_t) dt = \int_{\mathcal{S}} f(\mathbf{x}) \mu(d\mathbf{x}), \quad \text{for } \mu \text{ a.s. all initial conditions}$$

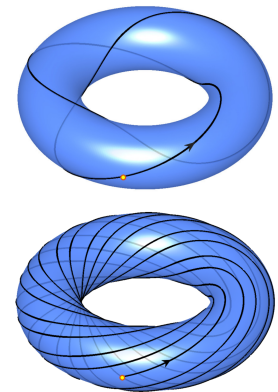
EXAMPLE TRANSLATION ON THE TORUS

- A translation on the 2-torus $\mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$ is a flow

$$\mathbf{R}_t(\mathbf{x}_1, \mathbf{x}_2) = (\mathbf{x}_1 + \omega_1 \mathbf{t}, \mathbf{x}_2 + \omega_2 \mathbf{t}) \pmod{1}$$

- Dichotomy of Orbits & Recurrence

- **Periodic Case:** If $\omega_1/\omega_2 = p/q \in \mathbb{Q}$, every orbit is a closed periodic curve
- **Quasi-Periodic / Ergodic Case:** If $\omega_1/\omega_2 \notin \mathbb{Q}$, every orbit is non-periodic and dense in $\mathbb{T}^2$



DYNAMICAL SYSTEMS – MORE DEFINITIONS

- **Unique Ergodicity** in the irrational case
 - There is a unique invariant probability measure: the standard 2D Lebesgue measure $\mu = dx_1 dx_2$
 - (R_t, μ) is ergodic
 - It is uniquely ergodic: for all continuous functions $f : \mathbb{T}^2 \rightarrow \mathbb{R}$, for all x :

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T f(R_t(x)) dt = \iint_{\mathbb{T}^2} f(x_1, x_2) dx_1 dx_2$$

THE WALKER CONSTELLATION POINT PROCESS AS A DYNAMICAL SYSTEM

- $\mathbf{x} = (\bar{\omega}, \bar{\theta})$: initial phases on the torus $[0, \frac{2\pi}{N_o}] \times [0, \frac{2\pi}{N_s}]$
- $\Psi(\mathbf{x}, t)$: Walker satellite point process at time t for initial $\mathbf{x}$
Full description of the **ephemerides** of the constellation
- $\Psi(\mathbf{x}, t)$ obtained from $\Psi(\mathbf{x}, 0)$ by a torus translation $\mathbf{R}(\mathbf{x}, t)$
- $\Psi(\mathbf{x}, t)$ is a deterministic measurable functional Ξ of $\mathbf{R}(\mathbf{x}, t)$
- Therefore, if $\mathbf{R}$ has an **invariant probability measure** $\mathcal{P}$ on S , then Ψ has an invariant probability measure as well, which is the push forward of $\mathcal{P}$ by Ξ , denoted by $\mathcal{P}_\Psi$
- If $\mathbf{R}$ is **ergodic** w.r.t. $\mathcal{P}$, so is Ψ w.r.t. $\mathcal{P}_\Psi$

WALKER DYNAMICS: INVARIANT PROB. MEASURE

- Let $\bar{\mathcal{Q}}$ denote the product of the two uniform distributions on $S = [0, \frac{2\pi}{N_o}] \times [0, \frac{2\pi}{N_o}]$
- Let $\bar{\mathcal{Q}}_\Psi$ denote the distribution of Ψ at time zero when $(\bar{\theta}, \bar{\omega})$ has distribution $\bar{\mathcal{Q}}$
- **Theorem [C.S. Choi, F. B., IEEE Veh. Tech. 25]**
The probability measure $\bar{\mathcal{Q}}$, on S , is invariant for R and the probability measure $\bar{\mathcal{Q}}_\Psi$, on $\mathcal{N}_r$, is invariant for Ψ

WALKER DYNAMICS: ERGODICITY, PERIODICITY

- Rotation speed ratio:

$$\rho = \frac{\bar{v}_\theta}{\bar{v}_\omega}$$

- **Theorem** [C.S. Choi, F. B., IEEE Veh. Tech. 25]

- If ρ is irrational, then the dynamical system
 - * $\mathbf{R}$ on $\mathbf{S}$ is **minimal and ergodic** w.r.t. $\overline{\mathcal{Q}}$
 - * Ψ on $\mathcal{N}_r$ is **ergodic** w.r.t. $\overline{\mathcal{Q}}_\Psi$
- If the ratio ρ is rational, then both $\mathbf{R}$ and Ψ are **periodic**

- When ρ is irrational, for $\overline{\mathcal{Q}}$ almost all $\mathbf{x}$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \mathbf{h}(\Psi(\mathbf{x}, t)) dt = \int \mathbf{h}(\psi) \overline{\mathcal{Q}}_{\Psi}(d\psi),$$

for all functions $\mathbf{h} : \mathcal{N} \rightarrow \mathbb{R}$ such that $\int |\mathbf{h}(\psi)| \overline{\mathcal{Q}}_{\Psi}(d\psi) < \infty$

- Stronger results for continuous functions
- When ρ is rational, the system is periodic. The long time average (e.g., associated with the ephemerides) of a given Earth user **does not** match averages w.r.t. the invariant distribution $\overline{\mathcal{Q}}_{\Psi}$

- Altitude determines the satellite speed
- The designer can choose an altitude making the constellation periodic to fixed Earth observers.
- One can claim that designers unaware of this dynamical system property have **almost surely** built constellations which are **aperiodic, minimal and ergodic**, and where long time empirical averages converge to the expectation w.r.t. the **invariant distribution $\mathcal{P}_\Psi$**

Recent extension to multi-altitude Walker constellations in
[Choi, FB, ArXiv June 26]

2. WALKER CONSTELLATION: DYNAMICS OF ROUTING TABLES

- Single altitude Walker Delta constellation
- Setting for routing:
 - Relays
 - * Satellites: $\mathcal{S}$
 - * Collection of gateway Earth stations: $\mathcal{G}$
 - Users: $\mathcal{U}$
- Graph:
 - Nodes: $\mathcal{V} = (\mathcal{U}, \mathcal{G}, \mathcal{S})$,
 - Edges: between two nodes that are LOS (different terrestrial nodes are NLOS)

2. WALKER CONSTELLATION: DYNAMICS OF ROUTING TABLES (*continued*)

- **Routing Tables $\{\Lambda_t\}$:** For each source – destination pair in $\mathcal{V}$, for all times t , Λ_t provides
 - A loop-free sequence of nodes in $\mathcal{V}$ in this graph whose interior nodes lie in $\mathcal{S} \cup \mathcal{G}$
 - The associated look angles from any node on this route to the next node
- Λ_t is assumed to be based on a time invariant rule based on Ψ_t e.g.
 - Shortest Euclidean distance path in the graph
 - Greedy routing

WALKER CONSTELLATION: NON PERIODICITY OF ROUTING TABLES

- **Theorem** [Choi, FB, ArXiv Oct. 26]

Assume that $\rho \notin \mathbb{Q}$. Then $\{\Lambda_t\}$, as a whole, is not periodic. In addition, no user terminal has a periodic look angle entry

- Proof based on the fact that $\{\Psi_t\}$ and $\{\Lambda_t\}$ are conjugate

- **Corollary**

For all values of ρ , the entropy of the routing table $\{\Lambda_t\}$ dynamical system is zero

NUMERICAL ILLUSTRATION

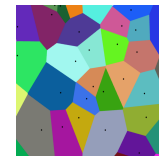
- The source is near Seoul and the destination near Paris
- **Fiber propagation RTT:**
 - Light in fiber propagates at $c/1.47$
 - An ideal fiber path along the great circle would have a propagation RTT of appr. **88.0 ms**
 - In practice, measurements give an RTT of about **247 ms**
- **360 satellite Walker constellation propagation RTT:**
constellation with $N_o = 24$ orbits, $N_s = 15$ satellites per orbit, inclination $\varphi = 53^\circ$, altitude 550 km, orbital period ≈ 95.6 min)
 - Under greedy routing (suboptimal), **the ergodic mean of the Walker propagation RTT is 72.1 ms**

3. HANDOVER DYNAMICS IN HAPS and UAV NETWORKS

- Dynamics of closest HAPS/UAV Association. **Poisson** →
Talk of Sanjoy Jhawar, Cox → Talk of Emanuele Mengoli
 - Initial station locations: Φ , a PPP of intensity λ on $\mathbb{R}^2$
 - i.i.d. motion directions : θ_i , e.g. $\text{Uniform}[-\pi, \pi)$, $\forall X_i$ in Φ
Results hold for all distributions of motion direction
 - Velocity vector: $V_i = v(\cos \theta_i, \sin \theta_i)$, where v constant
 - Locations at time t : $X_i^t := X_i + V_i \cdot t$

$$\Phi^t := \{X_i^t : X_i \in \Phi\}$$

Static user at the origin



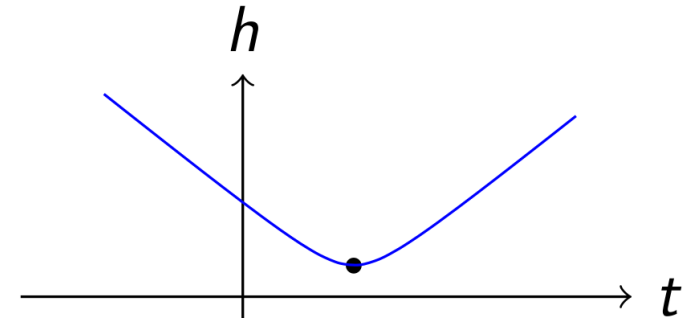
DISTANCE FUNCTION, BIRD RANDOM CLOSED SETS

- Let $\mathbf{X} \in \Phi$ with:
 - position
 $\mathbf{X} = (|\mathbf{X}| \cos \psi, |\mathbf{X}| \sin \psi)$
- Define $\alpha = \psi - \theta \bmod 2\pi$
 - velocity vector
 $\mathbf{V} = \mathbf{v}(\cos \theta, \sin \theta)$

-
- Distance function (of $\mathbf{t}$):
 - $\forall \mathbf{X} \in \Phi \rightarrow$ bird closed set:
 - $|\mathbf{X}^{\mathbf{t}}| = \sqrt{|\mathbf{X}|^2 + 2\mathbf{t}\mathbf{v}|\mathbf{X}| \cos \alpha + \mathbf{v}^2\mathbf{t}^2} \mathbf{C}_{\mathbf{X}} := \{(\mathbf{t}, \mathbf{f}_{\mathbf{v}}(\mathbf{X}, \mathbf{t})) : \mathbf{t} \in \mathbb{R}\}$
 - $:= \mathbf{f}_{\mathbf{v}}(\mathbf{X}, \mathbf{t})$

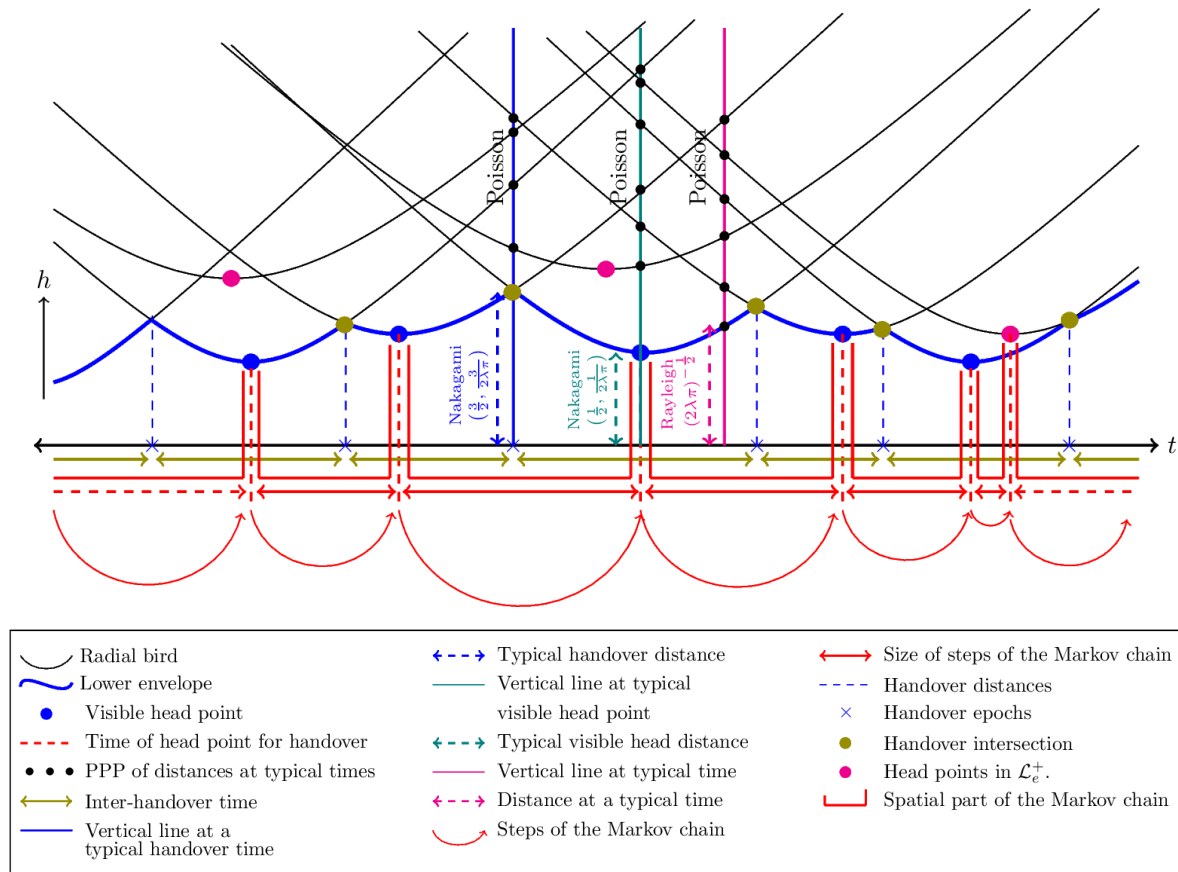
- Minimum of $\mathbf{f}_{\mathbf{v}}(\mathbf{X}, \mathbf{t})$,
 $(\mathbf{T}_{\mathbf{X}}, \mathbf{H}_{\mathbf{X}})$ (head point):

$$\left(-\frac{|\mathbf{X}|}{\mathbf{v}} \cos \alpha, |\mathbf{X}| |\sin \alpha| \right)$$



Branch of some hyperbola

OBJECTS OF INTEREST, SINGLE SPEED CASE



QUANTITATIVE PROPERTIES OF INTEREST

- **Frequency** of events (e.g. handovers)
- Event Statistics Based on **Palm Calculus**: exact statistics of
 - signal
 - interference
 - Shannon rate

at e.g. typical

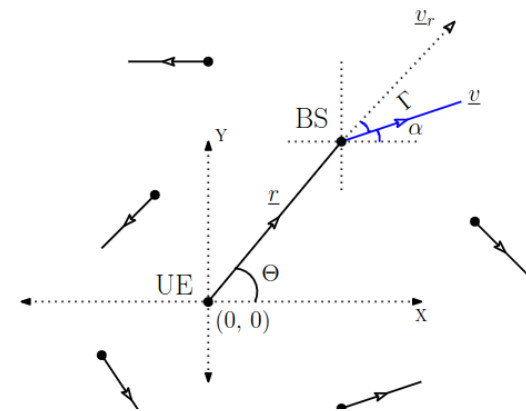
- handover epochs
- epoch when serving BS is at the closest
- epoch when first interferer is at the closest

[S. Jhavar and FB, ArXiv, → IEEE Tr IT, 26]

4. DOPPLER-SHANNON UAV/HAPS ASSOCIATION

Poisson UAVs/HAPS → Talk of Ashutosh Balakrishnan

- **Doppler shift:** change in signal frequency due to relative motion of transmitter and receiver
- Influenced by velocity along LoS v_r and by wavelength ρ
- **Doppler angle:** Γ , relative angle between LoS and direction of motion (DoM)



Doppler shift:

$$D = \frac{v_r}{\rho} = \frac{v \cos \Gamma}{\rho}$$

4. DOPPLER-SHANNON UAV/HAPS ASSOCIATION (*continued*)

- The effect of Doppler shift in frequency domain, is mapped in the time domain through the coherence time
- **Coherence Time**, (with μ an engineering constant):

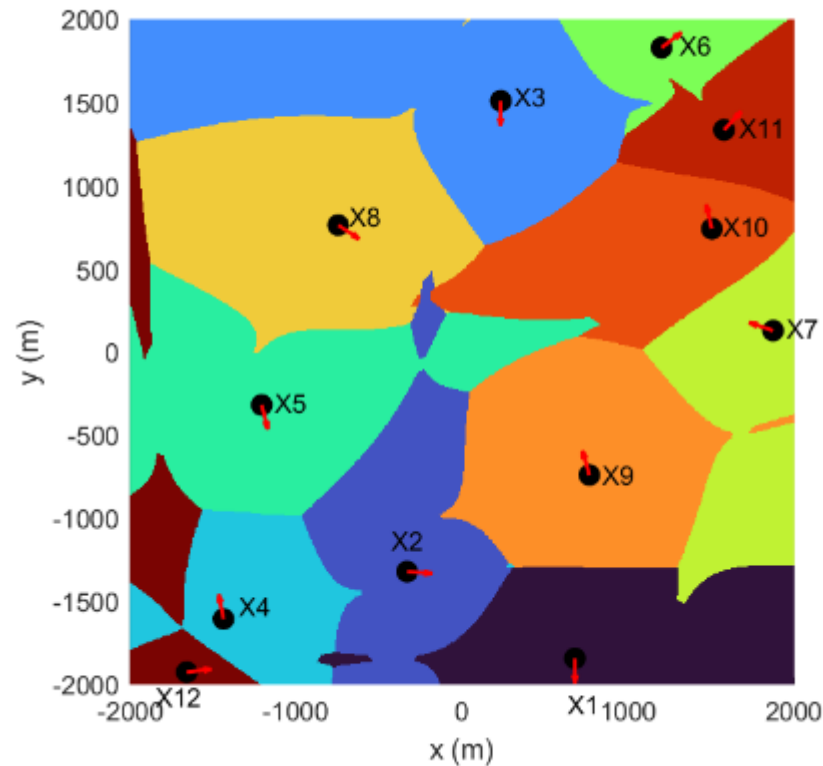
$$\mathbf{Tc}(\Gamma) = \frac{\mu}{\mathbf{D}} = \frac{\mu\rho}{\mathbf{v} \cos \Gamma}$$

- Duration for which the wireless channel remains flat, i.e., statistically i.i.d. In this duration, the wireless link is stable
- **Doppler-Shannon SNR Utility:**

$$\mathbf{U}_i = \min(\mathbf{Tc}(\Gamma_i), \zeta) \mathbf{B} \log_2 \left(1 + \frac{\mathbf{K}}{(\mathbf{h}^2 + \mathbf{R}_i^2)^{\alpha/2}} \right)$$

with ζ a constant

4. DOPPLER-SHANNON UAV/HAPS ASSOCIATION (*continued*)



Doppler–Shannon association → quantification of gain in spectral efficiency through spatial averages

Structure of the Presentation

- Non Terrestrial Communication Networks
- System Level Analysis Methodology
- Illustrations
- NTN Impacts
 - Astronomy
 - Chemistry of Higher Atmosphere
 - Near Space
- Ongoing Research, Actions

IMPACT ON OPTICAL ASTRONOMY

- The reflection of sunlight by satellites produces **streaks that contaminate telescopic images of the night sky**
- This has a **significant impact on sky surveys with large-field-of-view instruments**, in particular those searching for transient phenomena

IMPACT ON RADIO ASTRONOMY

- Radioastronomy is already very much disturbed, 24 hours a day, through interference from
 1. downlink transmissions
 2. on-board electronics at low frequencies

This threatens highly sensitive observations of radio sources and fundamental cosmological surveys

- Local radio sanctuaries accepted by some operators mitigate 1
- Even in this case, 2 disturbs sensitive radio observations

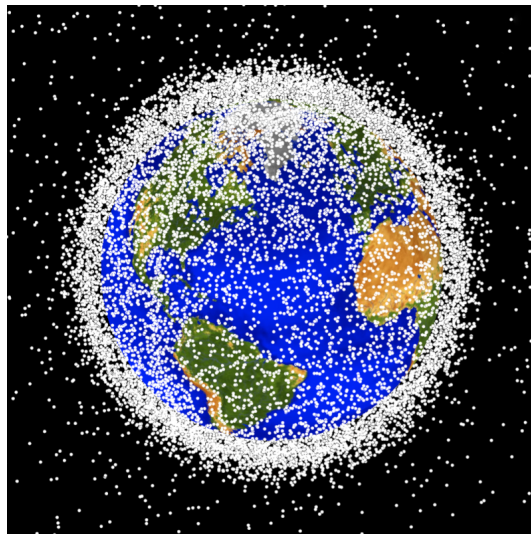
The multiplication of constellations will dramatically degrade observations if nothing systematic is done to mitigate their disturbances, even more if million scale constellations are deployed

IMPACT ON UPPER ATMOSPHERE CHEMISTRY

- The increase of launches leads to greater depositions of chemicals in the atmosphere:
 - during launch: **Black Carbon and other combustion products**
 - during satellite reentries: **Lithium, Aluminum, Titanium**
- **These chemicals accumulate in the mesosphere and stratosphere** Their residence times there is very long
- Their **impact on the upper atmospheric chemistry** and more specifically on the ozone layer are not well understood
- The impact of these launches and constellations on the **Earth's global radiative balance** should also be assessed

IMPACT ON NEAR SPACE

The current space debris situation



Orbiting debris
larger than 10 cm
NASA, 2022

More than 28,000 objects larger
than 10 cm currently tracked

Further orbiting debris:

- $\frac{1}{2}$ million of order 1 cm
- 100 million order 1 mm

Causes (beyond lack of care):

- Satellite collisions
- Use of anti-satellite weapons

SPACE TRAFFIC MANAGEMENT

- In 2009, a collision between an active US satellite and a dead Russian satellite generated over 1000 trackable debris staying in orbit
- The **number of maneuvers required for avoiding collisions** is already quite high: more than 100 000 in a single year of Starlink operation
- **Kessler Syndrome**: between 300 et 1100 km altitude, an uncontrolled augmentation of the number of satellites and debris could lead to
 - a multiplication of collisions
 - a **chain reaction** with exponential increase of the number of debris

Structure of the Presentation

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FURTHER AND ONGOING RESEARCH DIRECTIONS

- **ONGOING:** Definition and characterization of universal metrics for **Visibility, Coverage, Spectral Efficiency, Routing, Handovers**
- **FURTHER:**
 - Interplay between TN and NTN (with **Pierre Popineau (Univity)** and **Namyoon Lee (Korea University)**)
 - Sky Visibility (with **Ashutosh Balakrishnan (IMT)** and **Junse Lee (Sungshin University)**)
 - RIS enhanced NTN (with **Paul Rax (INRIA)** and **Junse Lee (Sungshin University)**)
 - SINR wall (with **Ali Khalesi (IPSA - LINCS)**)

FURTHER AND ONGOING RESEARCH DIRECTIONS (*continued*)

■ **BROADER: Impact Mitigation**

- Evaluation of risks
- Quantification of impacts
- Minimal constellation for prescribed service requirements
- **Scientific foundations for international regulation**

(PAST) AND ONGOING ACTIONS

- ERC NEMO: <https://project.inria.fr/ercnemo/>
- BPI project with Univity, Greenerwave, Onera (TP)
- Hubert Curien project with South Korea (MAE)
- Contribution to SRIA 25 and 26 (EC)
- Report of the Academie des Sciences 24 (AS)
- S7 Statement 26 (G7)
- Joining the Advisory Board of Directorate-General for Defence Industry and Space (EC)

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