

Seasonal statistics of Shannon rate in a dynamical Poisson-Voronoi cellular network

Sanjoy Kumar Jhawar

INFRES, Télécom Paris & MATHNET, INRIA Paris

Joint work with François Baccelli



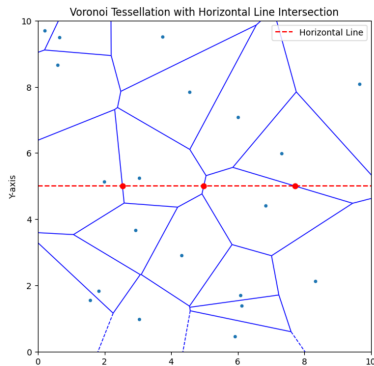
NTN Days, CNAM, Paris

14th October, 2026

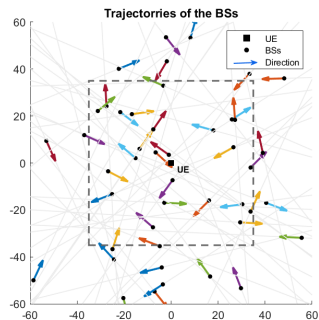
Outline

- 1 Mobility models
- 2 Motivation
- 3 Single-speed Poisson model
- 4 Seasons and seasonal statistics
- 5 Results on seasonal statistics

Mobility models



Trajectory of a mobile user along a straight line, among static stations



Dynamics of mobile stations: Straight line motion with random direction, in constant speed (Credit: Alex Beutel)

Motivation: non-terrestrial networks

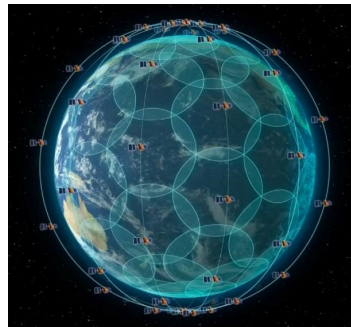
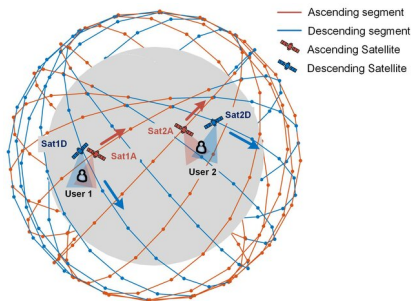


Figure: *Left.* Direct service to a user by satellites and handover phenomenon. (Picture credits: Chen, Giambene, Yang, Fan, Chen)

Right. Coverage areas of constellation of satellites. (Picture credits: IridiumWhere.com)

Distance function

- Let $\mathbf{X} \in \Phi$ with:
- Define $\alpha = (\psi - \theta) \bmod 2\pi$
- position $\mathbf{X} = (|\mathbf{X}| \cos \psi, |\mathbf{X}| \sin \psi)$
- velocity vector $\mathbf{V} = v(\cos \theta, \sin \theta)$

- Define the distance function of t

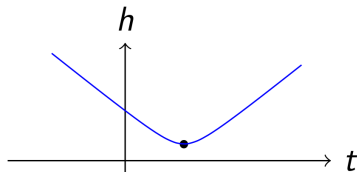
$$\begin{aligned}
 |X^t| &= |X + Vt| \\
 &= (|X|^2 + 2tv|X| \cos \alpha + v^2 t^2)^{\frac{1}{2}} \\
 &:= f_v(X, t)
 \end{aligned}$$

- Minimum of $f_v(X, t)$, $(\mathbf{T}_X, H_X)$, called head point, at

$$\left(-\frac{|X|}{v} \cos \alpha, |X| |\sin \alpha| \right)$$

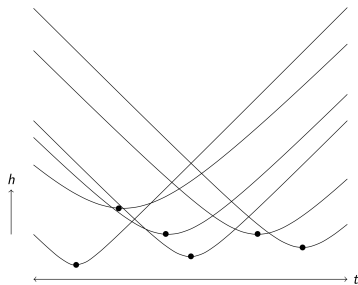
- $f_v(X, t) = (v^2(t - T_x)^2 + H_x^2)^{\frac{1}{2}}$

To each $X \in \Phi$, associate the bird closed set: $C_X := \{(t, f_v(X, t)) : t \in \mathbb{R}\}$



Branch of a hyperbola, looking like a bird, hence the name

Results on handover statistics

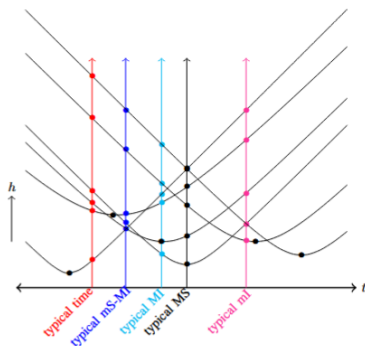


The following questions are answered in [Baccelli & J., 2026+](#)⁽¹⁾

- Handover frequency: $\lambda_V = \frac{4v\sqrt{\lambda}}{\pi}$
- Distribution of handover distance (H_V): Nakagami $(\frac{3}{2}, \frac{3}{2\lambda\pi})$

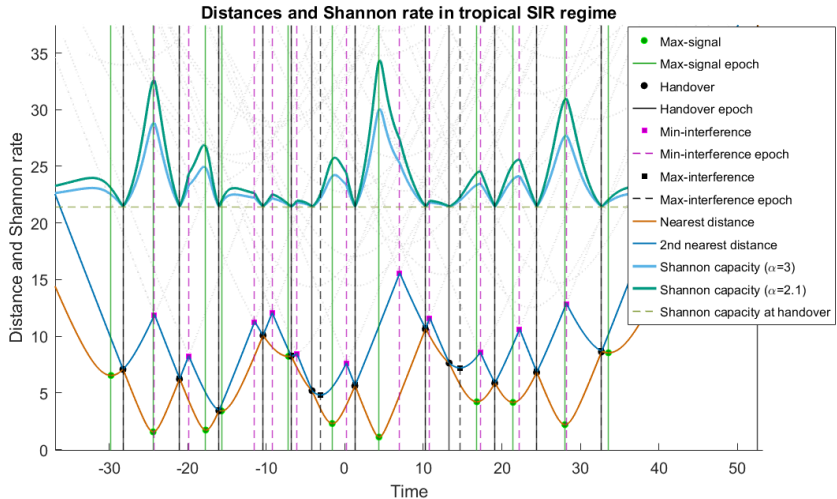
⁽¹⁾On a Class of Dynamical Poisson-Voronoi Tessellations, F. Baccelli and J, 2026+, arXiv:2511.15893

Seasons or typical epochs



- Identification of **seasons** or **typical epochs** of interest:
 - **Handover**: Serving BS and nearest interferer at equal distance
 - **Max-signal**: Serving BS at its closest distance
 - **Max-interference**: Nearest interferer at its closest distance
 - **Min-interference**: Nearest interferer at its farthest distance
- **Non-periodicity** of seasons
- **Frequency** of all these seasons
- **Palm probability measure** with respect to these seasons
- **Performance evaluation**: Coverage probabilities and Shannon rate

Seasons of Shannon rate (tropical case, without fading)



Seasonal statistics of Shannon rate: Objective

- Characterization of distances (serving BS and interferers) at these **seasons**
- **signal-to-interference-plus-noise-ratio**: $SINR = \frac{S}{N+I}$
- S : received signal power by the user at the corresponding season

$$S := \begin{cases} \rho H_v^{-\alpha} & \text{at typical mS-MI epoch,} \\ \rho H_s^{-\alpha} & \text{at typical MS epoch,} \\ \rho R_l^{-\alpha} & \text{at typical MI epoch,} \\ \rho R_x^{-\alpha} & \text{at typical ml epoch,} \\ \rho R^{-\alpha} & \text{at any typical time,} \end{cases}$$

- ρ : fading, α : path-loss exponent, I : interference, $N = \sigma^2$: thermal noise
- **Performance evaluation and comparison**⁽²⁾:
 - **Coverage probability**: $p^c(\tau, \mu, \lambda, \alpha) := \mathbb{P}(SINR > \tau)$,
 - **Average Shannon rate**: $\mathcal{R} := \mathbb{E}[B \ln(1 + SINR)] = B \int_0^\infty \frac{1}{1+z} \mathbb{P}(SINR > z) dz$

²Seasonal Statistics of Shannon Rate in a Dynamical Poisson-Voronoi Cellular Network, arXiv:2605.16560, (J. and F. Baccelli, 2026).

Performance results (J. & Baccelli, 2026+)

Theorem

The coverage probability with respect to Palm probability distribution of the typical mS-MI, MS and typical time epoch is given by

$$p^c(\tau, \mu, \lambda, \alpha) = \mathbb{E}_H^0 \left[e^{-\mu\tau H^\alpha \sigma^2} \mathcal{L}_{I_H}(\mu\tau H^\alpha) \right],$$

where $H \in \{H_v, H_s, R\}$.

Theorem

The coverage probability with respect to Palm probability distribution of typical ml and MI epoch is given by

$$p^c(\tau, \mu, \lambda, \alpha) = \mathbb{E}_{H,R}^0 \left[e^{-\mu\tau R^\alpha \sigma^2} \mathcal{L}_{I_H}(\mu\tau R^\alpha) \right],$$

where $R \in \{R_l, R_t\}$, $H \in \{H_l, H_t\}$.

- Expressions matches with results in the work by Andrews, Baccelli and Ganti (2011).

Closed forms: SINR regime

- PLE $\alpha = 2$: $p_*^c = 0$, in any season, i.e., $* \in \{\nu, s, l, \mathcal{I}, t\}$.
- PLE $\alpha = 4$ and $* \in \{\nu, s, t\}$:

$$p_*^c = \frac{1}{(1 + \tau)^{\mathbb{1}_{\{*= \nu\}}}} \mathbb{E}_{H_*}^0 \left[e^{-\mu \tau H_*^4 \sigma^2 - \pi \lambda H_*^2 \kappa(\tau, 4)} \right],$$

- $\kappa(\tau, \alpha) := \tau^{2/\alpha} \int_{\tau^{-2/\alpha}}^{\infty} \frac{1}{1+z^{\alpha/2}} dz$.
- Note, $\kappa(\tau, 2) = \infty$ and $\kappa(\tau, 4) = \tau^{\frac{1}{2}} \arctan(\tau^{\frac{1}{2}})$.
- PLE $\alpha = 4$ and $* \in \{l, \mathcal{I}\}$:

$$p_*^c = \frac{\tau^{-1/4}}{4} \int_0^\tau \frac{z^{-3/4}}{(1+z)^{1+\mathbb{1}_{\{*= \mathcal{I}\}}}} \mathbb{E}_{H_*}^0 \left[e^{-\mu z H_*^4 \sigma^2 - \pi \lambda H_*^2 \kappa(z, 4)} \right] dz.$$

Closed forms: SIR regime & scale invariance

- $\kappa(\tau, \alpha) := \tau^{2/\alpha} \int_{\tau^{-2/\alpha}}^{\infty} \frac{1}{1+z^{\alpha/2}} dz$.
- PLE $\alpha=2$, we have $p_*^c=0$, at any season, i.e., $* \in \{v, s, l, \mathcal{I}, t\}$, since $\kappa(\tau, 2) = \infty$.
- PLE $\alpha = 4$ and $* \in \{v, s, t\}$: $p_t^c = \left(1 + \tau^{\frac{1}{2}} \arctan(\tau^{\frac{1}{2}})\right)^{-1}$ (Andrews et al)

$$p_v^c = (1+\tau)^{-1} \left(1 + \tau^{\frac{1}{2}} \arctan(\tau^{\frac{1}{2}})\right)^{-3/2}, \quad p_s^c = \left(1 + \tau^{\frac{1}{2}} \arctan(\tau^{\frac{1}{2}})\right)^{-\frac{1}{2}}.$$

- PLE $\alpha = 4$ and $* \in \{l, \mathcal{I}\}$:

$$p_*^c = \frac{\tau^{-1/2}}{2} \int_0^\tau \frac{z^{-1/2}}{(1+z)^{1+\mathbb{1}_{\{*\equiv \mathcal{I}\}}}} \left(1 + z^{\frac{1}{2}} \arctan(z^{\frac{1}{2}})\right)^{-3/2-\mathbb{1}_{\{*\equiv \mathcal{I}\}}} dz.$$

- Relations with $p_t^c(\tau)$: $p_v^c(\tau) = \frac{(p_t^c(\tau))^{3/2}}{1+\tau}$, $p_s^c(\tau) = (p_t^c(\tau))^{1/2}$,

$$p_*^c(\tau) = \frac{\tau^{-1/2}}{2} \int_0^\tau \frac{z^{-1/2}}{(1+z)^{1+\mathbb{1}_{\{*\equiv \mathcal{I}\}}}} (p_t^c(z))^{3/2+\mathbb{1}_{\{*\equiv \mathcal{I}\}}} dz.$$

Comparison: coverage probabilities and Shannon rate

Theorem

The following comparison of the coverage probabilities holds true for any QoS $\tau \geq 0$:

- with or without fading: $p_v^c \leq p_t^c \leq p_s^c$ and $p_{\mathcal{I}}^c \leq p_l^c$.

Using $p_*^c(\tau) \equiv p^c(\tau, \mu, \lambda, \alpha) := \mathbb{P}^0(\text{SINR} > \tau)$, the average Shannon rate is

$$\mathcal{R}_* := \mathbb{E}^0[B \ln(1 + \text{SINR})] = B \int_0^\infty \frac{1}{1 + \tau} p_*^c(\tau) d\tau,$$

for $* \in \{v, s, l, \mathcal{I}, t\}$.

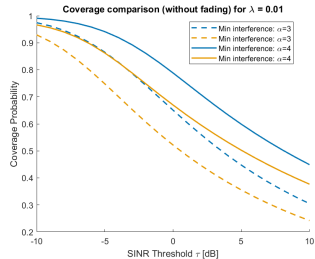
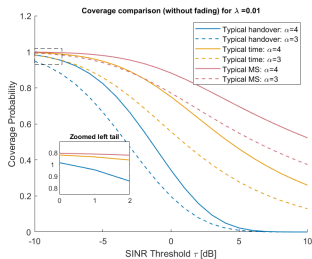
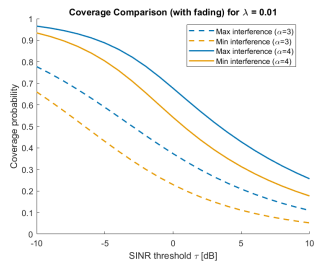
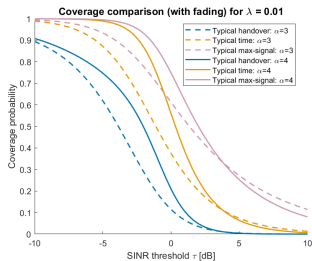
Corollary

The following comparisons of the average Shannon rates hold true:

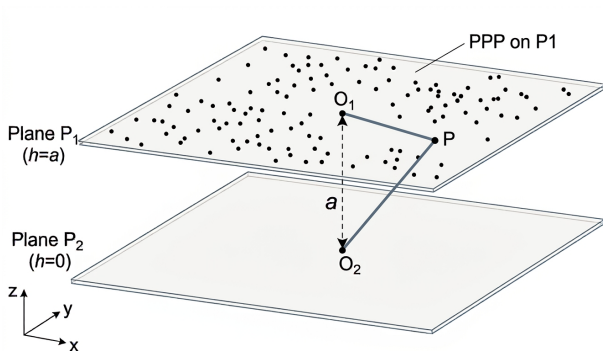
- with or without fading: $\mathcal{R}_v \leq \mathcal{R}_t \leq \mathcal{R}_s$ and $\mathcal{R}_{\mathcal{I}} \leq \mathcal{R}_l$
- with or without fading: $\mathcal{S}_v \leq_{\text{st}} \mathcal{S}_t \leq_{\text{st}} \mathcal{S}_s$ and $\mathcal{S}_{\mathcal{I}} \leq_{\text{st}} \mathcal{S}_l$,

where $\mathcal{S}$ denotes the SINR.

Comparison of coverage probabilities (with and without fading)



Extension to 3D models: UAVs and HAPS



- User at O_2 , with its zenith at O_1
- Radial distance from zenith: $r = |PO_1|$
- Altitude: $a = |O_1O_2|$
- Attenuation function: $\ell(r, a) = (r^2 + a^2)^{-\alpha/2}$.

Coverage probabilities

Theorem

The coverage probabilities with respect to Palm probability distribution of epoch of interest, in the non-tropical and tropical case are as follows:

- ① *for typical mS-MI, MS and typical time epoch*

$$p^c(\tau, \mu, \lambda, \alpha) = \mathbb{E}_H^0 \left[e^{-\mu\tau(H^2 + a^2)^{\alpha/2}\sigma^2} \mathcal{L}_{I_H}(\mu\tau(H^2 + a^2)^{\alpha/2}) \right],$$

where $H \in \{H_v, H_s, R\}$.

- ② *for typical ml and MI epoch*

$$p^c(\tau, \mu, \lambda, \alpha) = \mathbb{E}_{H,R}^0 \left[e^{-\mu\tau(R^2 + a^2)^{\alpha/2}\sigma^2} \mathcal{L}_{I_H}(\mu\tau(R^2 + a^2)^{\alpha/2}) \right],$$

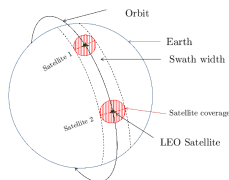
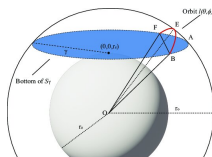
where $R \in \{R_l, R_x\}$, $H \in \{H_l, H_x\}$.

Future work

Consider the constellation of low earth orbit (LEO) satellites

- Obstacles:

- 1 Mobility of satellites
- 2 Sky visibility region with respect to user
- 3 Coverage area or the spherical cap on the Earth surface, of a LEO satellite moving on an orbit



- Questions:

- 1 Handover frequency between user-satellite and other types
- 2 Handover statistics
- 3 Seasonal statistics of Shannon rate
- 4 Multi-hop routing

References

- J. G. Andrews, F. Baccelli, and R. K. Ganti, A Tractable Approach to Coverage and Rate in Cellular Networks, in *IEEE Transactions on Communications*, vol. 59, no. 11, pp. 3122–3134, 2011.
- F. Baccelli and S. Zuyev, Stochastic Geometry Models of Mobile Communication Networks, in *Frontiers of Queuing Theory*, Volume 2, pp. 227–244, 1997.
- F. Baccelli and J., On a Class of Dynamical Poisson-Voronoi Tessellations, arXiv:2511.15893, 2026+.
- C. S. Choi and F. Baccelli, A Novel Analytical Model for LEO Satellite Constellations Leveraging Cox Point Processes, *IEEE Transactions on Communications*, 73(4): 2265–2279, 2025.
- J. and F. Baccelli, Seasonal Statistics of Shannon Rate in a Dynamical Poisson-Voronoi Cellular Network, arXiv:2605.16560, 2026. (Accepted in Trans-IT).

Thank you for your attention!

Questions?