

Handover Frequency in Cox LEO NTN

How predictable are handovers in a moving satellite network and what does that mean for network design?

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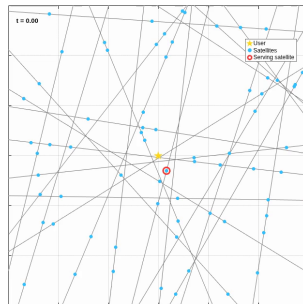
Handovers of a static user are driven by satellite motion

- LEO-600 [1]: $v \approx 7.6$ km/s. A satellite stays above 30° elevation for at most 4.1 min.
- A static user thus changes serving satellite repeatedly.
- Three quantities characterise this process:

How often? handover rate λ_{HO} $\rightarrow$ signalling load

How far? distance at a handover $\rightarrow$ link margin

How long? inter-handover time $\rightarrow$ timers, bursts

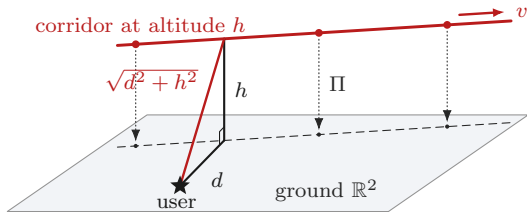


Red ring: the serving satellite.

Research question

How predictable are handovers in a moving satellite network and what does this predictability imply for network design?

UAVs at a common altitude: reduction to the plane

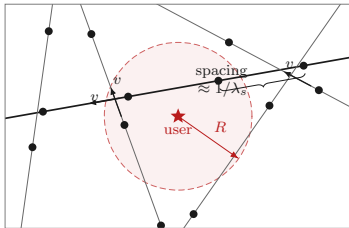


- The vertical projection preserves lines, speeds and Poisson laws.
- Pythagoras restores the distance: $\|X\|^2 = \|\Pi X\|^2 + h^2$.
- $r \mapsto \sqrt{r^2 + h^2}$ is increasing: the nearest UAV in the air is the nearest on the ground.

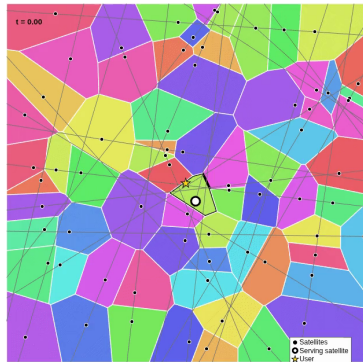
Reduction to the plane

Same serving UAV, same handover instants. The distance at a handover, $\tilde{Z}$ in the plane, becomes $\sqrt{\tilde{Z}^2 + h^2}$ at altitude h .

Model: Poisson orbits, Poisson satellites, three parameters



- λ_ℓ : orbit intensity. Mean number of orbits within distance R of the user: $2\lambda_\ell R$.
- λ_s : satellites per unit length of orbit, Poisson. Gaps are $\text{Exp}(\lambda_s)$: mean spacing $1/\lambda_s$.
- v : common speed. The user (star) is served by the satellite of its Voronoi cell, the nearest one.



Key property

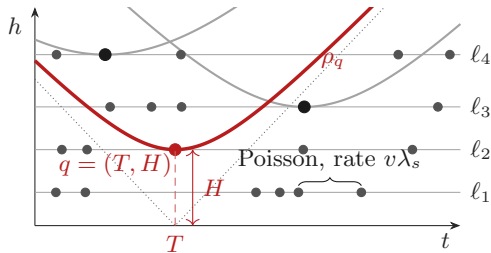
The orbits are fixed. Only the satellites move along them.

Each pass as a point (T, H) : the head process

- A pass is described by its closest-approach time T and distance H [2].
- Its distance to the user:
 $\rho(t) = \sqrt{H^2 + v^2(t - T)^2}$.

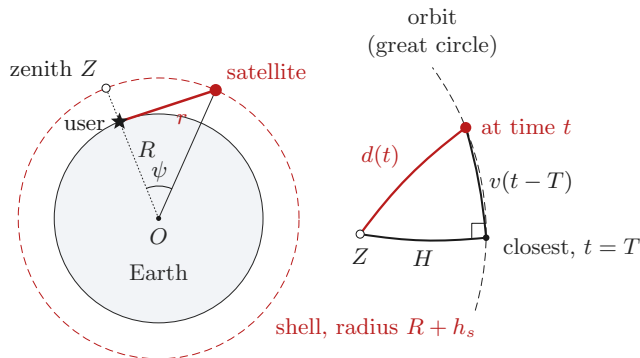
Head process $\mathcal{H} = \{(T, H)\}$

- Poisson times of rate $v\lambda_s$ along the orbits.
- Uniform intensity $2v\lambda_s\lambda_\ell$ in (t, h) .
- Independent of the orbit directions.



Each point (T, H) is the bottom of its pass's distance curve (three shown) on the orbit.

LEO shell: the planar model holds within 0.1% above 30°

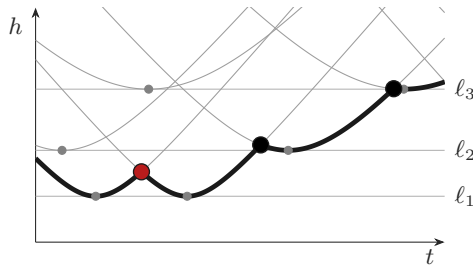


- Nearest in slant range = nearest along the shell.
- A pass forms a right spherical triangle.
- Near the zenith:
 $d^2 \approx H^2 + v^2(t - T)^2$, the planar curve.

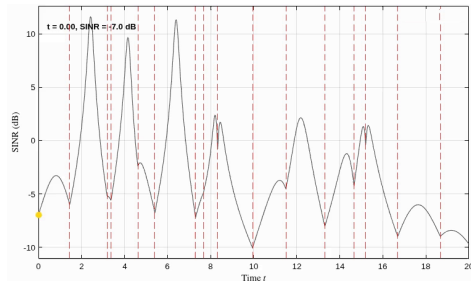
Worst-case error

- Satellites above 30° (LEO-600 [1]): $\leq 0.075\%$.
- Down to 10°, a larger cap: $\leq 0.32\%$.

The serving distance is a lower envelope. Handovers are its kinks



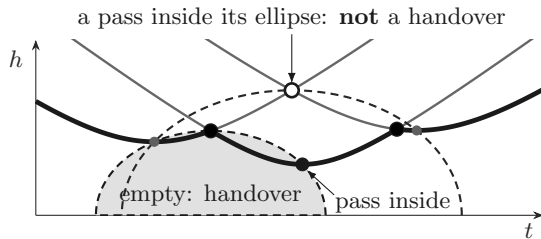
Thick: the serving distance.



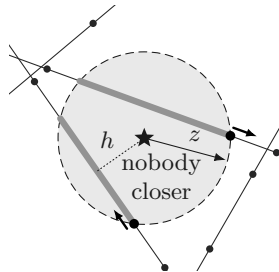
Simulated SINR (free-space). Dashed: handovers.

- The serving distance is the **lower envelope** of the distance curves. A handover is a kink.
- **Red**: to the next satellite of the same orbit. **Black**: to a satellite of another orbit.
- At a handover both satellites are equidistant, hence equally strong.

Handover criterion: an empty half-ellipse



Handover iff no pass (T, H) with
$$v^2(T - m)^2 + H^2 < z^2.$$



The same handover in the sky:
no satellite within z .

Method

- $\lambda_{\text{HO}} = \text{crossing pairs} \times \mathbb{P}(\text{empty half-ellipse})$, for **same-orbit** and **cross-orbit** pairs.
- Explicit void probability: $\mathbb{P}(\text{no drone within } z) = \exp(\pi \lambda_\ell z M_1(2\lambda_s z))$.
 $M_1 = L_1 - I_1$: modified Struve minus modified Bessel [3].

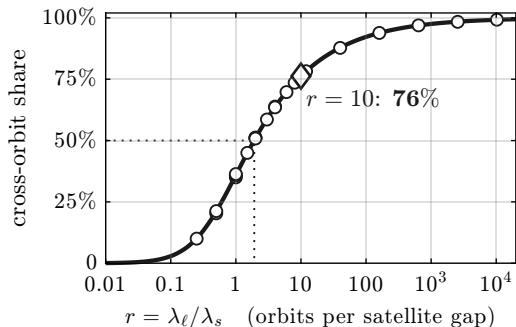
Result 1: the handover rate obeys a scaling law in λ_ℓ/λ_s

$$\lambda_{\text{HO}} = \underbrace{v\lambda_s}_{\text{same orbit}} \left(1 + \underbrace{C(\lambda_\ell/\lambda_s)}_{\text{cross orbit}} \right)$$

- $C(r)$: ratio of cross-orbit to same-orbit handovers (backup).

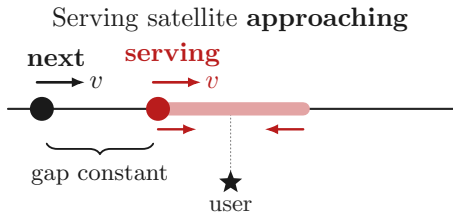
Implication

- The speed scales every rate and leaves the composition unchanged.
- The composition depends on $r = \lambda_\ell/\lambda_s$ only. Equal shares at $r \approx 2$.

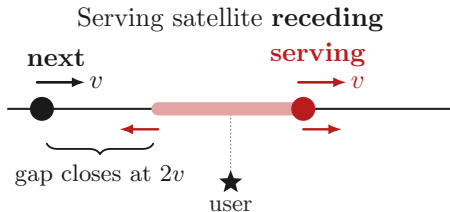


Line: theory $C/(1+C)$. Circles: simulation at 22 pairs $(\lambda_\ell, \lambda_s)$, 4000 realisations each.

Same-orbit handovers occur at rate $v\lambda_s$ for every user



no same-orbit handover



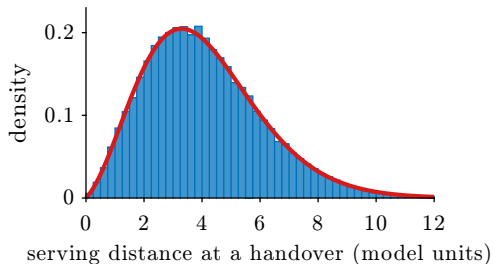
handovers at rate $2v\lambda_s$

- Handover: the next satellite enters the shaded stretch.
- Approaching: never. Receding: at rate $2v\lambda_s$.
- By symmetry, each phase lasts half of the time: $\lambda_{so} = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 2v\lambda_s = v\lambda_s$.

Consequence

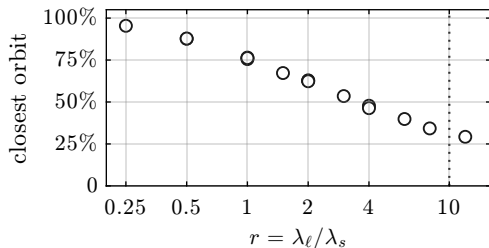
- Identical for every user.
- User differences stem from **cross-orbit** handovers only.

Result 2: handover distance and closest-orbit dominance



Distance at a handover ($r = 10$): theorem vs simulation.

- Small r : handovers along the dominant closest orbit. Large r : between several orbits.



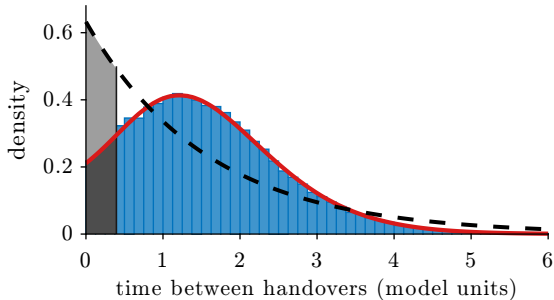
The probability that at a random instant the serving satellite is on the closest orbit.

Implication

- The margin at handover is set by the geometry of the closest orbits.

Result 3: inter-handover times are not memoryless

- **Not memoryless:** the density peaks at $t > 0$, the exponential at $t = 0$.
- **Short gaps are rare:** 10 % below $\frac{1}{4}$ of the mean gap ($1/\lambda_{\text{HO}}$), against 22 %.
- Cause: a newly acquired satellite is usually still approaching.
- $\mathbb{P}_{\Psi}^0(\tilde{T} > t) = \mathbb{P}(\text{no pass in sweep area})$ (backup).



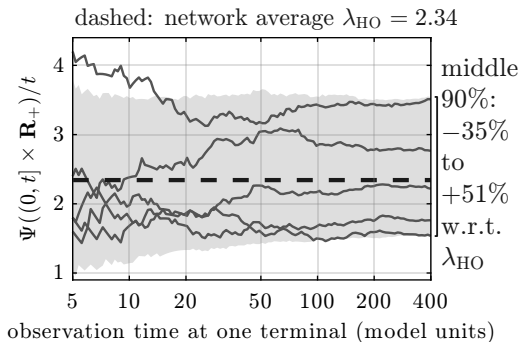
Histogram: simulation, solid line: the theorem, dashed: exponential, same mean. Filled: gaps shorter than $\frac{1}{4}$ of the mean gap (dark: theorem, light: exponential).

Implication

- The mean rate does not characterise the timing.

Result 4: the handover process is stationary but not ergodic

Does a long measurement at one location yield the network handover rate?



5 simulated realisations (5, 25, 50, 75 and 95 % positions) among the 300 realisations. Shaded: 90 % of the realisations.

No: stationary, but **not ergodic**.

- Time shifts move the satellites, not the orbits $\mathcal{L}$.
- $\frac{1}{t} \Psi((0, t] \times \mathbb{R}_+) \rightarrow \lambda_{\text{HO}}(\mathcal{L})$, a random limit.
- Network mean: $\mathbb{E}[\lambda_{\text{HO}}(\mathcal{L})] = \lambda_{\text{HO}}$ (Result 1).
- The dispersion is entirely **cross-orbit**.

Implication

- Estimating λ_{HO} requires many locations.

Conclusions

1. **Handover frequency obeys a scaling law.**

$\lambda_{\text{HO}} = v\lambda_s(1 + C(\lambda_\ell/\lambda_s))$: the speed sets the time scale. The ratio λ_ℓ/λ_s sets the split between **same-orbit** and **cross-orbit** handovers.

2. **Handover conditions are characterised in distribution.**

Closed-form laws for the distance at a handover and for the inter-handover time. The latter is not memoryless.

3. **The handover process is not ergodic.**

A single location does not reveal the network average. Spatial averaging is required.

The objective is not simply to count handovers. It is to understand their geometry, predict their timing and identify what those statistics can and cannot, tell us about network performance.

Future work: the sphere itself (periodic passes), several shells and speeds, multi-hop routing.

Thank you



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References

- [1] 3GPP, “Solutions for NR to support non-terrestrial networks (NTN),” 3rd Generation Partnership Project, Technical Report TR 38.821, V16.2.0, Release 16, Mar. 2023, LEO-600: Tables 4.2-2 and 6.1.3.2-1.
- [2] F. Baccelli and S. K. Jhavar, “On a class of dynamical Poisson–Voronoi tessellations,” *arXiv preprint arXiv:2511.15893*, 2025.
- [3] “NIST digital library of mathematical functions,” 2025, release 1.2.4 of 2025-03-15. [Online]. Available: <https://dlmf.nist.gov/>

Backup: drones, the lift in formulas

A drone on a corridor at altitude h whose ground trace is at distance d from the user:

$$\|X(t)\|^2 = d^2 + h^2 + v^2(t - T)^2 = \rho_{2D}(t)^2 + h^2.$$

- **Common altitude.** Every arc is lifted by the same increasing map $\rho \mapsto \sqrt{\rho^2 + h^2}$, so the lowest arc stays the lowest: the handover times are those of the plane (same λ_{HO} , same law of $\tilde{T}$) and $\tilde{Z}_{\mathbb{R}^3} = \sqrt{\tilde{Z}^2 + h^2}$. Void probability: $\mathbb{P}(\text{no drone within } z) = e^{\pi\lambda_\ell\zeta M_1(2\lambda_s\zeta)}$ with $\zeta = \sqrt{z^2 - h^2}$, for $z > h$.
- **Altitude per corridor h_ℓ .** Heads $(T, \sqrt{d_\ell^2 + h_\ell^2})$: the hyperbolas keep their slopes $\pm v$, so the same construction applies with this new law of heights; the handover instants now differ from the plane.

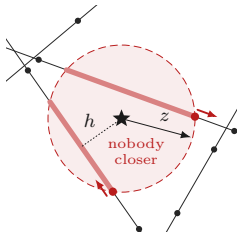
Backup: the sphere, how the numbers are computed

- **Visible cap.** With $a = R + h_s$, a satellite at central angle $\psi = d/a$ from Z is above elevation ε iff $\psi \leq \psi_{\max} = \arccos(\frac{R}{a} \cos \varepsilon) - \varepsilon$ (law of sines in the triangle Earth centre–user–satellite); $d_{\max} = a \psi_{\max}$ is the cap radius along the shell. Inputs: 3GPP LEO-600 [1], $h_s = 600$ km, $\varepsilon = 30^\circ$ (target, Table 6.1.3.2-1) or 10° (minimum, Table 4.2-2); $R = 6371$ km (mean Earth radius, our assumption), so $a = 6971$ km, $R/a = 0.9139$. Slant range $s = \sqrt{R^2 \sin^2 \varepsilon + 2Rh_s + h_s^2} - R \sin \varepsilon$.

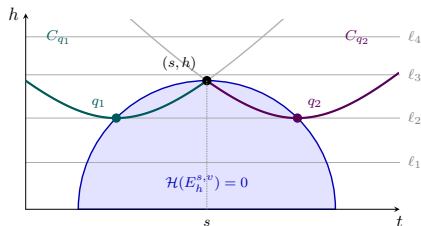
ε	$\psi_{\max}$	$d_{\max} = a \psi_{\max}$	s
30°	$\arccos(0.7915) - 30^\circ = 7.68^\circ = 0.1340$ rad	$6971 \times 0.1340 = 934$ km	1075 km
10°	$\arccos(0.9000) - 10^\circ = 15.84^\circ = 0.2764$ rad	$6971 \times 0.2764 = 1927$ km	1932 km (Table 4.2-2)

- **Speed.** $v = \sqrt{GM/a} = 7.56$ km/s: a zenith pass above 30° lasts $2d_{\max}/v = 4.1$ min.
- **Planar error.** With $b = v(t - T)$, expanding $\cos \frac{d}{a} = \cos \frac{H}{a} \cos \frac{b}{a}$ to fourth order gives $d^2 = H^2 + b^2 - \frac{H^2 b^2}{3a^2} + O(d^6/a^4)$, so $\rho = \sqrt{H^2 + b^2}$ overestimates d by at most $(d/a)^2/24$ (worst case $H = b$): 0.075% at 30° , 0.32% at 10° (exact computation).
- **Heights.** An orbit with pole n is at distance H from Z with $\sin(H/a) = |\langle n, Z \rangle|$. Isotropic poles: $\langle n, Z \rangle \sim \mathcal{U}[-1, 1]$ (Archimedes), so H has density $a^{-1} \cos(H/a)$: uniform near Z , as for Poisson lines (N planes $\leftrightarrow \lambda_\ell = N/(2a)$), within $1 - \cos(d_{\max}/a)$: 0.9% at 30° , 3.8% at 10° .
- **What the plane misses.** Passes repeat with the orbital period and two arcs may cross more than once per period. Walker constellations have two local directions (ascending, descending) rather than isotropic orbits.

Backup: nobody closer, the void probability



In the sky: $\Phi^m(B(O, z)) = 0$,
where Φ^m is Φ at time m .



In $\mathbb{H}_+$: $\mathcal{H}(E_z^{m,v}) = 0$,
 $E_z^{m,v} = \{(x, y) : v^2(x - m)^2 + y^2 < z^2\}$.

Lemma (void probability) — PGFL of the Poisson lines

$$\mathbb{P}(\mathcal{H}(E_z^{m,v}) = 0) = \mathbb{E}\left[\prod_{\ell \in \mathcal{L}} e^{-\lambda_s \mu_1(\ell \cap B(O, z))}\right] = \exp\left(-2\lambda_\ell \int_0^z (1 - e^{-2\lambda_s \sqrt{z^2 - h^2}}) dh\right) = e^{\pi \lambda_\ell z M_1(2\lambda_s z)}$$

$M_1 = L_1 - I_1$: modified Struve minus modified Bessel [3]

Backup: why two cases, the second factorial moment of $\mathcal{H}$

A handover is a functional of a **pair** of heads:

$$\Psi = \sum_{q_i, q_j \in \mathcal{H}}^{\neq} \delta_{(m_{ij}, z_{ij})} \mathbf{1}_{T_j < T_i} \mathbf{1}_{\mathcal{H}(E_{z_{ij}}^{m_{ij}, v})=0}.$$

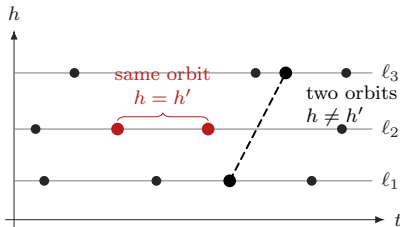
Second-order Campbell–Mecke formula:

$$\mathbb{E} \left[\sum_{x, y \in \mathcal{H}}^{\neq} f(x, y, \mathcal{H}) \right] = \iint \mathbb{E}_{\mathcal{H}}^{x, y} [f(x, y, \mathcal{H})] \alpha_{\mathcal{H}}^{(2)}(dx, dy).$$

$\mathcal{H}$ is Cox, directed by $\Lambda_{\mathcal{H}|\mathcal{L}} = v\lambda_s dt \otimes \sum_{\ell \in \mathcal{L}} \delta_{H_\ell}$:

$\alpha_{\mathcal{H}}^{(2)} = \mathbb{E}[\Lambda_{\mathcal{H}|\mathcal{L}} \otimes \Lambda_{\mathcal{H}|\mathcal{L}}]$ splits over $\ell = \ell'$ and $\ell \neq \ell'$,

$$\alpha_{\mathcal{H}}^{(2)} = \underbrace{2v^2\lambda_s^2\lambda_\ell dt dt' dh \delta_h(dh')}_{\alpha_{\text{so}}^{(2)}: \text{one orbit}} + \underbrace{(2v\lambda_s\lambda_\ell)^2 dt dt' dh dh'}_{\alpha_{\text{co}}^{(2)}: \text{two orbits (Mecke)}}.$$



Two-point Palm kernels

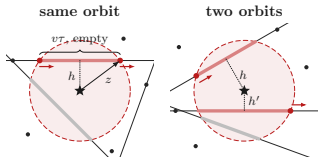
$\mathbb{E}_{\text{so}}^{x, y}$: $\mathcal{H}$ under $\mathcal{L} + \delta_{\ell_h}$;

$\mathbb{E}_{\text{co}}^{x, y}$: $\mathcal{H}$ under $\mathcal{L} + \delta_{\ell_h} + \delta_{\ell_{h'}}$.

Hence $\lambda_{\text{HO}} = \lambda_{\text{so}} + \lambda_{\text{co}}$ and

$$\mathbb{P}_{\Psi}^0 = \frac{\lambda_{\text{so}}}{\lambda_{\text{HO}}} \mathbb{P}_{\Psi, \text{so}}^0 + \frac{\lambda_{\text{co}}}{\lambda_{\text{HO}}} \mathbb{P}_{\Psi, \text{co}}^0.$$

Backup: the handover-rate theorem



Rate = pairs of passes that meet, weighted by the probability that nobody is closer:

- the other orbits: $e^{\pi\lambda_\ell z M_1(2\lambda_s z)}$ (Lemma);
- the orbit(s) of the pair: their chords in $B(O, z)$ are empty.

τ : time between the two closest passes; z : their meeting distance (backup).

Theorem (handover rate) $\lambda_{HO} = \lambda_{so} + \lambda_{co}$

$$\lambda_{so} = \underbrace{2v^2\lambda_s^2\lambda_\ell}_{\alpha_{so}^{(2)}} \int_{\mathbb{R}_+^2} \underbrace{e^{\pi\lambda_\ell z M_1(2\lambda_s z)}}_{\text{other orbits}} \underbrace{e^{-v\lambda_s \tau}}_{\text{chord } v\tau} d\tau dh,$$

$$\lambda_{co} = \underbrace{2(2v\lambda_s\lambda_\ell)^2}_{\alpha_{co}^{(2)}, h' < h} \int_0^\infty \int_0^h \int_0^\infty \underbrace{e^{\pi\lambda_\ell z M_1(2\lambda_s z)}}_{\text{other orbits}} \underbrace{e^{-2\lambda_s \max\{v\tau, \frac{h^2-h'^2}{v\tau}\}}}_{\text{two chords}} d\tau dh' dh.$$

Backup: the scaling $\lambda_{\text{HO}} = v\lambda_s(1 + C(r))$ and its two limits

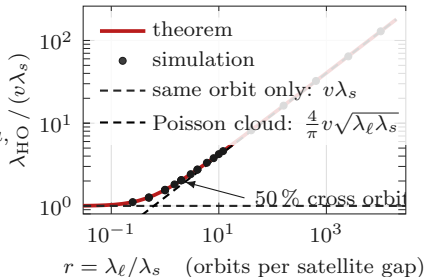
Proposition (scaling). Substitute $h = u/\lambda_s$, $h' = u'/\lambda_s$, $\tau = s/(v\lambda_s)$ in the rate theorem:
 $\lambda_{\text{HO}} = v\lambda_s(1 + C(r))$, $r = \lambda_\ell/\lambda_s$, with

$$C(r) = 8r^2 \iint\limits_{0 < u' < u, s > 0} e^{\pi r \zeta M_1(2\zeta)} e^{-2 \max\{s, \frac{u^2 - u'^2}{s}\}} ds du' du,$$

$\zeta = \sqrt{u^2 + \frac{1}{4}(s - \frac{u^2 - u'^2}{s})^2}$. By the Palm decomposition:

$$\mathbb{P}_\Psi^0(\text{cross orbit}) = \frac{\lambda_{\text{co}}}{\lambda_{\text{HO}}} = \frac{C(r)}{1 + C(r)}.$$

- $r \rightarrow 0$ (few crowded orbits): $C(r) \rightarrow 0$.
- $r \rightarrow \infty$, $\lambda_\ell \lambda_s$ fixed: numerically
 $\lambda_{\text{HO}} \rightarrow \frac{4}{\pi} v \sqrt{\lambda_\ell \lambda_s}$ (Poisson–Voronoi [2]).



The rate in units of $v\lambda_s$: one curve for all $(\lambda_\ell, \lambda_s, v)$. Dots: the 22 simulated pairs.

Backup: the distance at typical handover

The distance mark of a pair is its meeting distance, so Campbell–Palm on the two parts of $\alpha_{\mathcal{H}}^{(2)}$, with the integrands $p_h, p_{h,h'}$ of the rate theorem, gives

$$\mathbb{P}_{\Psi}^0(\tilde{Z} \in A) = \frac{1}{\lambda_{\text{HO}}} \left[\int \mathbf{1}_A(z_h(\tau)) p_h(\tau) \alpha_{\text{so}}^{(2)}(d\tau, dh) + \int \mathbf{1}_A(z_{h,h'}(\tau)) p_{h,h'}(\tau) \alpha_{\text{co}}^{(2)}(d\tau, dh, dh') \right].$$

Changing variables from (τ, h) , resp. (τ, h, h') , to the meeting distance z :

$$f_{\tilde{Z}}(z) = -\frac{2\pi v \lambda_s^2 \lambda_{\ell}}{\lambda_{\text{HO}}} z M_0(2\lambda_s z) e^{\pi \lambda_{\ell} z M_1(2\lambda_s z)} + \frac{4v \lambda_s^2 \lambda_{\ell}^2}{\lambda_{\text{HO}}} z^2 e^{\pi \lambda_{\ell} z M_1(2\lambda_s z)} \tilde{J}(2\lambda_s z),$$
$$\tilde{J}(a) = \frac{4}{a} \int_0^{\pi/4} \tan \xi (e^{-a \sin 2\xi} - e^{-2a \cos^2 \xi}) d\xi - \pi M_0(a) \left(1 + \frac{\pi}{2} M_1(a)\right).$$

$M_{\nu} = L_{\nu} - I_{\nu}$ [3]. Each term integrates to its share: $\int f_{\tilde{Z},\text{so}} = \lambda_{\text{so}}/\lambda_{\text{HO}}$, $\int f_{\tilde{Z},\text{co}} = \lambda_{\text{co}}/\lambda_{\text{HO}}$.

Backup: inter-handover time, Laplace form and swept length

Equivalently to the survival form (Theorem 2.23 of the paper):

$$\mathbb{E}_{\Psi}^0[e^{-s\tilde{T}}] = 1 - \frac{s}{\lambda_{\text{HO}}} \int_0^{\infty} e^{-st} \left[2v^2 \lambda_s^2 \lambda_{\ell} \int_{\mathbb{R}_+^2} p_h(\tau) F_{\text{so}}(t) d\tau dh + (2v\lambda_s \lambda_{\ell})^2 \int_{\mathbb{R}_+^3} p_{h,h'}(\tau) F_{\text{co}}(t) d\tau dh dh' \right] dt.$$

Palm kernels (the integrands of the rate): $p_h(\tau) = e^{\pi\lambda_{\ell} z_h M_1(2\lambda_s z_h)} e^{-v\lambda_s \tau}$,
 $p_{h,h'}(\tau) = e^{\pi\lambda_{\ell} z_{h,h'} M_1(2\lambda_s z_{h,h'})} e^{-2\lambda_s \max\{v\tau, |h^2 - h'^2|/(v\tau)\}}.$

Swept length, for the active pair $q_- = (-\tau, h')$, $q_+ = (0, h)$ meeting at (m, z) and $u \geq 0$:

$$x_u = m + u, \quad r_u(y) = \sqrt{x_u^2 + \frac{h^2 - y^2}{v^2}}, \quad H_u = \sqrt{h^2 + v^2 x_u^2},$$
$$L_u(y) = [x_u + r_u(y) - \max\{0, x_u - r_u(y)\}]_+, \quad \Delta_t(y) = L_t(y) - L_0(y) = \mu_1(\mathcal{S}_t \cap \ell_y).$$

$F_{\text{so}}(t) = A_t e^{-v\lambda_s \Delta_t(h)}$, $F_{\text{co}}(t) = A_t e^{-v\lambda_s (\Delta_t(h) + \Delta_t(h'))}$,
 $A_t = \exp(-2\lambda_{\ell} \int_0^{H_t} (1 - e^{-v\lambda_s \Delta_t(y)}) e^{-2\lambda_s \sqrt{(z^2 - y^2)_+}} dy)$. Evaluated by quasi-Monte Carlo; it matches 4 000 simulated realisations within sampling noise (exact KS 0.0058, noise level 0.0075).

Backup: the Poisson limit recovers the classical handover rate

Path: $\lambda_\ell = 0.5 k$, $\lambda_s = 0.05/k$, $v = 3$, so $\lambda_s \lambda_\ell = 0.025$ is fixed, $r = 10 k^2$ and $\varepsilon = 1/k \rightarrow 0$: denser orbits, fewer satellites on each. The satellites tend to a Poisson cloud of intensity $\lambda_s \lambda_\ell$, and

$$\lambda_{\text{HO}} \rightarrow \frac{4}{\pi} v \sqrt{\lambda_s \lambda_\ell} = 1.2732 \times 3 \times \sqrt{0.025} = 1.2732 \times 3 \times 0.1581 = 0.6040,$$

the classical rate of a user crossing a Poisson–Voronoi network at speed v [2].

k	λ_ℓ	λ_s	r	λ_{HO} (theorem)	simulation ($\pm$ s.e.)	theorem vs limit
1	0.5	0.05	10	0.6323	0.6343 ± 0.0024	+4.70%
2	1	0.025	40	0.6170	0.6150 ± 0.0019	+2.16%
4	2	0.0125	160	0.6102	0.6108 ± 0.0017	+1.04%
8	4	0.00625	640	0.6070	0.6078 ± 0.0016	+0.51%
16	8	0.003125	2 560	0.6055	0.6025 ± 0.0015	+0.25%
32	16	0.0015625	10 240	0.6047	0.6047 ± 0.0014	+0.12%

The excess over the limit halves when k doubles: about $4.7\% \times \varepsilon$, so the orbits add about 5% at $\varepsilon = 1$ ($r = 10$). Why the limit: for a Poisson cloud the head process is the same whether the stations or the user move.